Searching For Long-Lived Binary Inspirals In Future Gravitational-Wave Detectors

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Abstract

Third-generation gravitational wave detectors like Einestine Telescope (ET) bring forth a new opportunity to look further back in history than ever before. With greater sensitivity, ET is able to uncover many overlapping continuous wave signals. This included signals from neutron star inspirals that fall within the low-frequency band. Typical data analysis methods like Matched Filtering become computationally expensive and less time efficient when tackling overlapping continuous wave signals. However, the Generalized Frequency Hough is an alternative to Matched Filtering. By using Matlab simulated neutron star merging system, the Generalized Frequency Hough method is tested for its efficiency at detecting overlapping signals. It was found that at an amplitude of order 1e-23 and higher, the efficiency of detecting overlapping signals is approximately 1.0 or 100% . We find that the Generalized Frequency Hough is not only able to detect overlapping signals with great success but also recover the initial parameters and duration of the overlapping continuous wave signals.

I. Introduction

With the construction of third-generation Gravitational Wave (GW) detectors, gravitational wave signals will be captured more than ever before. Longer-lasting continuous wave signals will be detected in the lower frequency bands, uncovering GW signals from the early Universe. However, because of the sensitivity of these instruments, it becomes increasingly difficult to retrieve these signals from very noisy data samples. Because of this, novel approaches to data analysis must be used to tackle increasingly precise data samples within next-generation GW detectors.

Match Filtering is a data analysis method used to recover signals from gravitational wave data and is the most popular method used for current GW data. As the name suggests, this method matches templates to an unknown signal. A template is a known signal or pattern. Templates are used to predict unknown signals, by matching the peaks or nodes of the template to the unknown signals. Many templates are needed to compare with a signal before a prediction is made. By correlating all of the templates a signal is then recovered. To date, this has been the more efficient way of detecting gravitational wave signals. However, with increasingly sensitive instruments that can register signals within lower frequency bands, Match Filtering becomes computationally expensive and less efficient. Since signals are remaining for longer times in the detector, more templates need to be used. Additionally, this method fails when overlapping signals are introduced. Therefore, match filtering will not be an ideal method when retrieving signals from

An alternative to Match Filtering is proposed in this report. The Frequency Hough Transform(FHT) is an alternative that is less computationally heavy and efficient for detecting gravitational wave signals. In this report, we will discuss the efficiency of the Hough Transform at retrieving overlapping signals. In this report, we will analyze the efficiency of this method and explain why it should be used to retrieve overlapping signals from ET.

II. Background

i Gravitational Waves and Neutron Stars

Gravitational waves (GW) are emitted by anything that accelerates and has asymmetrical rotation. This asymmetry produces a quadrupole mass moment that has a non-zero third derivative, necessary for a gravitational wave signal emission. This means that something as small as your car can emit gravitational waves. However, such systems have very small amplitudes and only cause noise within larger systems' GW signals. The sensitivity of current and next-generation GW detectors retrieves signals for everyday vibrations like walking or birds. As a result, GW signals are embedded in a lot of noise.

Additionally, there are different types of gravitational waves. These types are dependent on the time each GW spends in the detector. These types are burst, stochastic, compact binary, and continuous, the longest, where burst is the shortest and continuous is the longest. Continuous waves last hours to days within the detector whereas burst only last a fraction of a second.

Asymmetrical rotating neutron star mergers are a candidate source of continuous gravitational waves (CGW). When two neutron stars rotate about one another, eventually colliding it is said to be a neutron star coalescence. Neutron star mergers lose energy over time as they emit gravitational waves which can be computed with the chirp mass. The chirp mass determines the leading-order orbital evolution of the system as a result of energy loss from emitting gravitational waves. Where μ_c is the chirp mass,m_1 is the first mass and m_2 is the second mass.

$$\mu_c = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1 + m_2)^{\frac{1}{5}}} \tag{1}$$

ii Generalized Frequency Hough

A Hough transform is a method that is used to detect simple shapes such as straight lines, circles, or ellipses. By utilizing an image space and parameter space, points on the image space map onto lines in the parameter space. Because of its versatility, this method is utilized to detect continuous gravitational waves that follow a power law where $\dot{f}$ is the spin down, k is a constant, f is the frequency and n is the breaking index of the star.

$$\dot{f} = -kf^n \tag{2}$$

To understand the frequency evolution we take the integral of the above equation and arrive at:

$$f = \frac{f_0}{(1 + k(n-1)f_0^{n-1}(t - t_0))^{\frac{1}{n-1}}} \tag{3}$$

Where f_0 is the initial frequency and t is time. The proportionality constant k Because it is known that frequency and spin down have a power law relationship, the Generalized Hough transform can be adapted to predict the path and therefore be a means of detecting continuous gravitational waves. This is done using:

$$x = f^{1-n} \tag{4}$$

The Frequency Hough Transform takes lines in a time-frequency plane and transforms them into points in a frequency spin-down plane where the tine-frequency plane is the image space and the spin-down frequency plane is the parameter space. Below is an example of what the input and output of the GFH look like where the input is lines in the image space and the output points in the parameter space.

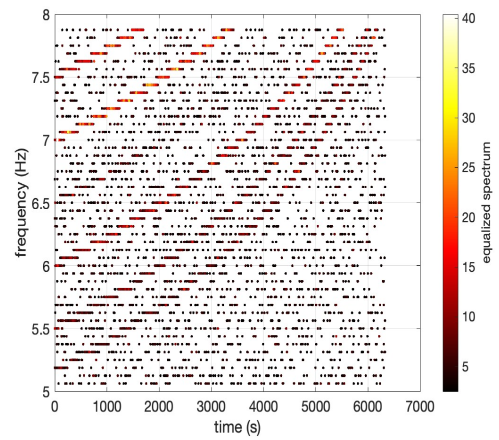


Figure 1: A Peak map where time is on the x-axis and frequency is on the y-axis. Here six signals are injected into the Hough transform, each represented by a line. The color identifies the intensity. Peak maps represent the input to the Hough transform

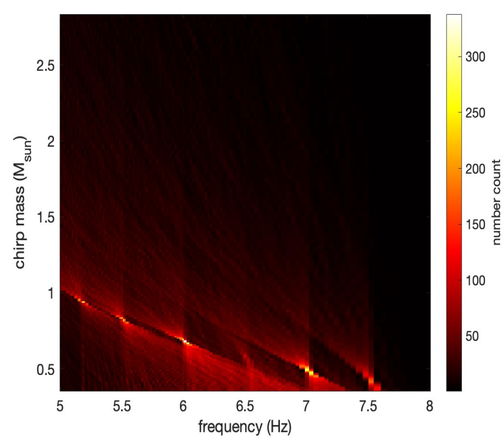


Figure 2: A Hough Map where the frequency is on the x-axis and chirp mass is on the y-axis. Here the six signals appear again and are shown by the bright white points. The color represents the intensity of the signal. Hough maps represent the output of the Hough transform.

III. Method

i Injecting Signals

In order to simulate neutron star coalescence, signals first must be injected. This process inputs a short fast Fourier Transform database (SFDB), which is created by dividing the data into chunks of length T_{FFT}. This T_{FFT} must be chosen to be small enough to confine the signal power into a frequency bin. The T_{FFT} length is computed as follows. Where $\dot{f}$ is the spin down.

$$T_{FFT} = \frac{1}{\sqrt{2\dot{f}}} \tag{5}$$

With the T_{FFT} length we can calculate the number of chunks as follows, where T_{obs} is the total length of observations.

$$N = \frac{T_{obs}}{T_{FFT}} \tag{6}$$

Then white noise is added to the SFDB to simulate a continuous gravitational wave signal. By embedding the signal, it resembles signals that ET will retrieve. Additionally, through this method, multiple overlapping signals may be injected. After choosing the initial parameters the simulation will calculate a random time after the first injection to inject the second, third, and so on signal.

ii Fast Foyer Transform and Hough Transform

Before the signal can go through a Hough transform it first needs to go through a Fast Foyer transform (FFT). Computing the FFT is as follows:

$$F(x) = \int_{-\infty}^{\infty} f(x)e^{-x}dt \tag{7}$$

The FFT is taken for each chunk so that each FFT can produce its own column in the Hough map. Each chunk is then stacked atop one another and assigned a color based on its power. When these are stacked on top of one another the colors change with the intensity which is why in Figure 1 each time-frequency coordinate pair has a color assigned to it. The Fourier transform is responsible for creating the peak map (Figure 1).

Next, the Frequency Hough transforms points in the time-frequency plane of the detector to lines in the frequency-spindown plane. of the source. Because the signals within the peak map are nonlinear. The generalized form of the FH must be used. Equation 4 demonstrates the generalized form. However, before applying the generalized transform, the coordinate systems for Equation 4 must be converted to become linear. This transformation is:

$$x = \frac{1}{f^{n-1}} : x_0 = \frac{1}{f_0^{n-1}} \tag{8}$$

After this transformation is applied we can substitute this into the frequency evolution equation (Equation 3) and arrive at:

$$x = x_0 + (n-1)(t - t_0)k \tag{9}$$

Points from the frequency-time plane can now be plotted as lines in the frequency-chirp mass plane. When applying the GH transform to every FFT, w each pixel in the graph will light up depending on the magnitude of intensity.

iii Simulation Parameters

In order to determine the efficiency of the Hough transform, many simulations needed to be performed with a variety of parameters. Parameters were selected with three main intentions: have a desirable T_{FFT} length, have accurate chirp-mass, and amplitude of a neutron star binary system, and simulations must be able to run on a home device. Chirp mass is

To ensure these, we first selected physical parameters, amplitude, and chirp mass. The chirp mass was set to 1.15 M_{sun} as it is the average chirp mass of a neutron star binary system. For each number of injected signals, at an amplitude of 1e-22, 1e-23, and 1e-24 meters were selected. This was done to ensure we could analyze the efficiency of signals of varying strengths.

Then, the frequency band was selected to find the desired T_{FFT} length. The desired length must be large enough so that each signal remains in the detector for a long enough period of time to constitute as a CW. Recall from Equation 5, the T_{FFT} length is dependent on the frequency. You will notice that the frequency and T_{FFT} have an inverted relationship. Therefore, the smaller the maximum frequency the larger the T_{FFT} length. Of course, we must still keep in mind that CW is typically within the lower frequency bands therefore we must limit the range from 2 to 20 Hz.

Lastly, the simulation had to run on a home or local device that contains limited ram and storage for SFDBs. Therefore two additional cuts were made. Instead of running 100 simulations per amplitude per number of signals, resulting in 30k simulations only 50 simulations were conducted per amplitude per number of signals. This resulted in 15,000 simulations. Additionally, because of limited storage on the local device, only 120 SFDBs were stored. To ensure the simulations would not take days to run, a frequency range of 4-7hz was selected. This range limited the number of SFBDs used and minimized the time per simulation to 3-4 seconds, and had a T_{FFT} length of 32 where each bin is 1k seconds within the detectors. This meant that for each 3-second simulation, each injected signal would remain in the detector for approximately 8 hours. Finally, the number of injections ranges from 1 to 10, for each varying number of injections 150 simulations were conducted. 50 simulations for amplitude 1e-22, 50 for amplitude 1e-23, and 50 for amplitude 1e-24. This was done because ET will be sensitive to overlapping signals and therefore the GHF was tested to ensure it can recover multiple overlapping signals. Each round of simulations would take my home device approximately 15 hours.

iv Data Extraction

All simulations were conducted in Matlab. In order to conduct data analysis on the simulations, each iteration within the 1500 simulations needed to be stored in a tabular fashion. In order to store the simulated data, a program was created to tabularize the parameters of each signal and compose a CSV file. The user input parameters, injected parameters, and found parameters were stored. (frequency, amplitude, number of injected signals, number of recovered signals $T_F FT$ length, chirp mass, distance away). Once composed into a CSV file, the text file was readable in Python where the data analysis occurred.

IV. Results

The purpose of this investigation is to measure the efficiency of the GFH method in retrieving overlapping injected signals of neutron star coalescence. In doing so efficiency is calculated as such:

$$E_f = \frac{N_{found}}{N_{inj}} \tag{10}$$

Where E_f is efficiency $N_f ound$ is the number of signals recovered after the frequency Hough transform. In order to compute the error for each point the equation for standard deviation of a binomial distribution was utilized. this is done because detecting a signal has a binomial distribution. Much like a coin flip the signal is either detected or not detected. Recall:

$$\sigma = \sqrt{\frac{p(1-p)}{n}} \tag{11}$$

Where p is the probability or in our case efficiency and n is the number of trials or total number of injections.

i Found Criteria

The criteria for detecting a signal is the bin distance away. We vary the criteria to be 3 bins away, 4 bins away, and 5 bins away. Below are two examples of distance distributions, on the left for 8 injected signals and on the right for 6 injected signals. From these graphs, it can be seen that the majority of the found signals are from amplitudes 1e-22 and 1e-23 lie within 3 bins away. However, for amplitude 1e-24, this is not the case. Visually we anticipate for approximately half of the found signals lie within 3 bins away.

ii Data Analysis

From Figure 4 we see four different graphs that display a number of signals on the x-axis and efficiency on the y-axis. Figure(4a) represents the efficiency of detecting signals when the found criteria are 3 bins away. As predicted from the distance distribution graph the majority of signals with amplitudes 1e-22 and 1e-23 were detected. The efficiencies for each of these amplitudes begin to decrease at 8 injected signals. For signals with an amplitude of 1e-24, the efficiency is much smaller beginning at approximately 0.5 and decreasing to 0.4 as the number of signals increases. This could be due to the fact that an amplitude of 1e-24 is a very weak signal and becomes harder to detect.

Figure (4b), which represents the efficiency of a signal being found with the criteria of being 4 bins away, also shows that the signals with amplitudes 1e-22 and 1e-23 are recovered with very high efficiencies. Again we see that the efficiency begins to drop when 8 or more overlapping signals are injected. The efficiency from signals with amplitude 1e-24 becomes greater when the criteria are within 4 bins away beginning at an efficiency of 0.8

Figure (4c) illustrates the efficiency versus a number of signal graphs when the detection criteria are 5 bins away. At 5 bins away all all signals with an amplitude of 1e-22 and 1e-23 are detected. The efficiency of signals with amplitude 1e-24 increases again to approximately an efficiency of 0.8. Similarly, Figure (4d) has the same trend for signals with amplitude 1e-22 and 1e-23. In Figure (4d) the efficiency of signals with amplitude 1e-24 raised to 0.9.

By comparing all of these graphs, we can note that the mergers with amplitude 1e-24 were farther, and therefore creating the bin range allowed them to be detected. Additionally, signals with amplitude 1e-22 and 1e-23 lie within a range of 5 bins. This is noteworthy because it allows us to determine at what din range the signals are most likely to be detected.

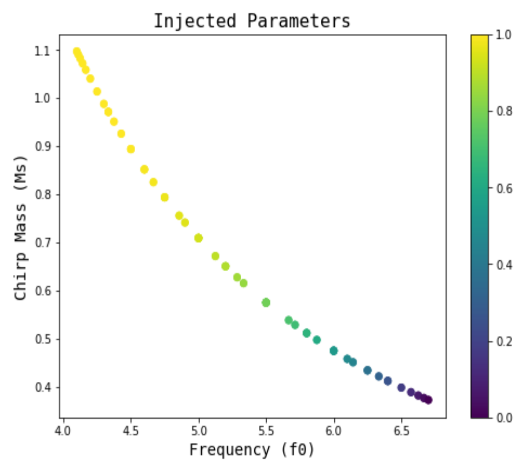


Figure 5: Injected Parameters of signals

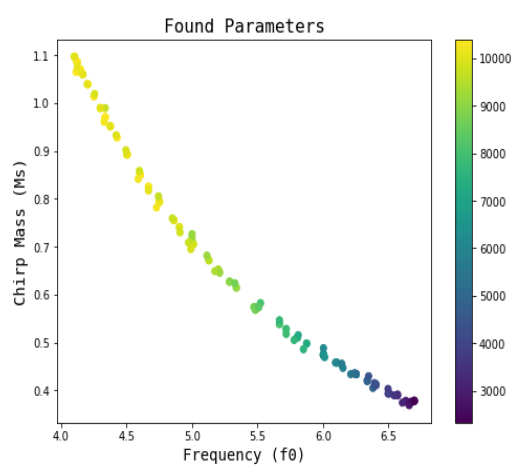
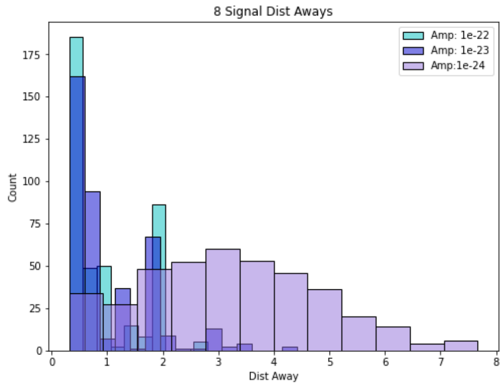
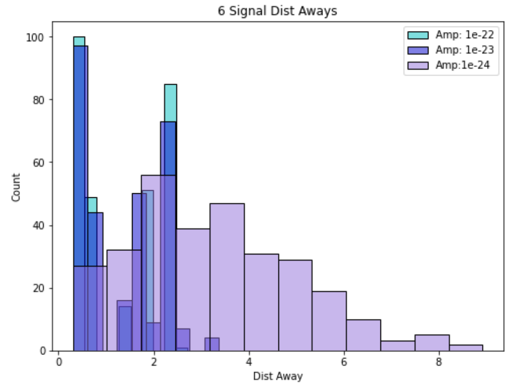


Figure 6: Found parameters of detected signals.

Not only is it important to detect injections but

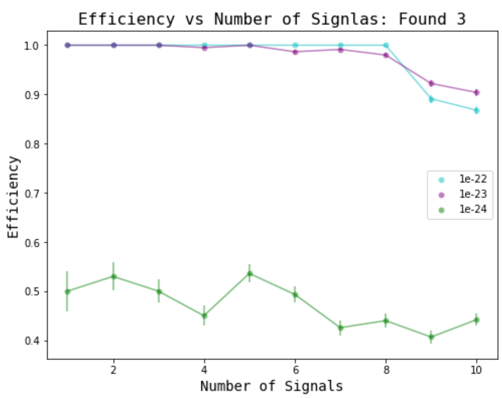


(a)

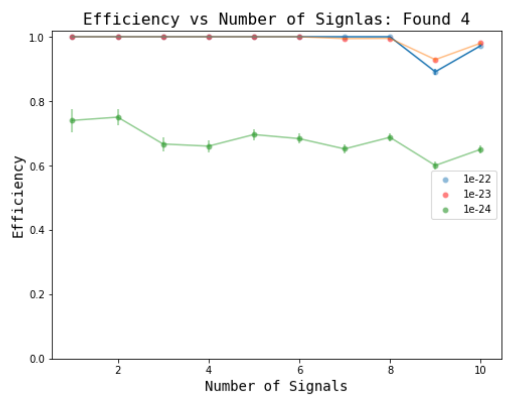


(b)

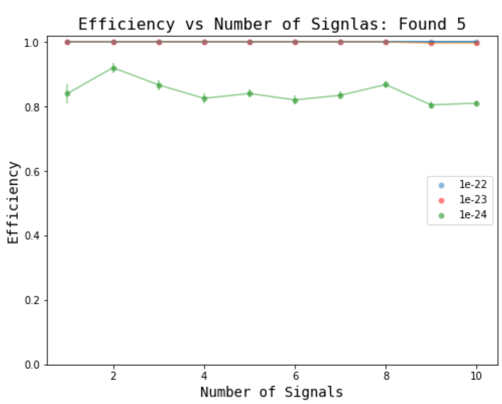
Figure 3: (a)Distance away distribution for 8 injected signals. (b) Distance away distribution for 6 injected signals.



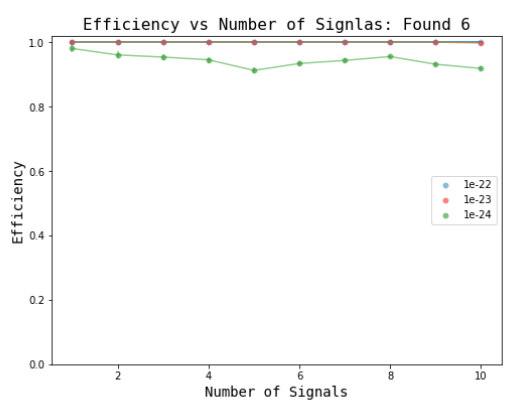
(a)



(b)

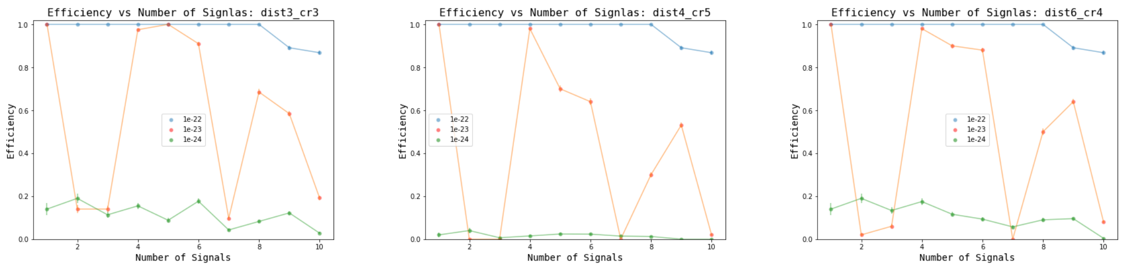


(c)



(d)

Figure 4: Efficiency versus Number of Signals for varying found criteria.



(a) (b) (c)

Figure 7: Primary Result: Efficiency graphs with additional of critical ratio criteria. (a) has the criteria of 3 bin distance and a critical ratio of 3. (b) has the criteria of 4 bins away and a critical ratio of 5. (c) has the criteria of 6 bins away and a critical ratio of 4.

also to retrieve the signals with the same parameters. In Figure 5 and Figure 6, there are two graphs that illustrate the injected and found parameters. The time within the detector is calculated as follows:

$$t_{merge} = \frac{3}{8}\frac{f^{-\frac{8}{3}} - f_0^{-\frac{8}{4}}}{k} \tag{12}$$

Where k is computed by:

$$k = -\frac{96}{5}\pi^{-\frac{8}{3}}(\frac{G\mu}{c^3})^{\frac{5}{3}} \tag{13}$$

When substituting k into equation 12 we can find the time using:

$$t = \frac{5}{250}(\pi f - f_0)^{-\frac{8}{3}}(\frac{G\mu M_{sun}}{c^3})^{-\frac{5}{3}} \tag{14}$$

Alternatively, the times can be calculated individually and duration can be computed as such.

$$t_{merge} = t - t_0 \tag{15}$$

From these graphs we can note two things, signals with a low frequency and high chirp mass have a longer duration, and signals with a larger frequency and low chirp mass have a shorter duration. Additionally, the two are comparable, we notice that the two curves are very similar. This suggests that the simulation is working effectively in recognizing the parameters of a signal.

Lastly, we analyzed that recovered signals had the same parameters and their respective inject signals. From Figures 5 and Figure 6, we see that the two are close to identical. This demonstrates that the GFH is able to detect the signal and retrieve its initial parameters effectively.

V. Conclusion

The purpose of this investigation was to prove that the Generalized Frequency Hough was a suitable alternative to matched filtering for detecting continuous gravitational waves produced by neutron star coalescence. In this report, we explain what the Generalized Frequency Hough is, and why it is a suitable and efficient solution for detecting CGW. The GFH is a method that is able to map lines that follow a power law to points. This means that signals will be displayed as a bright point in Hough Maps. The GFH also is able to derive signal parameters.

When implementing overlapping signals with varying amplitudes the efficiencies for larger amplitudes were close to 100%. This meant that even at distance criteria of 3 bins signals with amplitudes, the GFH was able to detect the majority of injected signals regardless of whether they were overlapping. Additionally, when varying the distance criteria, the efficiency for all amplitudes increased to be almost 100% efficiency, where signals from amplitude 1e-22 and 1e-23 had 100% efficiency and signals with amplitude 1e-24 have 90% efficiency at a criteria of 6 bins away.

The GFH is an optimal alternative to Matched Filtering for upcoming data from ET because it is face, efficient, and effective at identifying signals from CGW produced by neutron star mergers. The reason why CGW is important to detect with efficiency is because of ET ability to have new levels of precision in the lower frequency bands where CGW are produced.

i Further Work

Another crucial part of detecting GW signals is the strength of the signal in addition to the distance away. The critical ratio allows us to check for false alarms. If a signal has a low critical ratio this means that the signal is weak and would not be realistic to detect. Implementing this detection criterion into the GFH will lower the efficiency of signals at all amplitudes. Figure 7 shows the preliminary results of the analysis of how different critical ratio criteria affect the efficiency of detecting overlapping signals. These graphs are preliminary because of the errors within the retrieved signals with amplitude 1e-23. This is known because the efficiency should not drop suddenly as it does here. This notifies me that there must be an error in either the data extraction or efficiency computation. For efficiency to be near zero, very few signals must be detected and meet both the distance and critical value criteria. The source of the error is currently being looked into and will show clarity with further analysis.

VI. Acknowledgements

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