Complete merger detection using the Lunar Gravitational Wave Antenna

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The Lunar Gravitational Wave Antenna (LGWA) is a proposed Moon-based gravitational wave observatory capable of detecting waves in the deciHertz frequency by observing the vibrational eigenmodes of the Moon. It has the potential to do groundbreaking science, particularly in conjunction with the future Laser Interferometer Space Antenna (LISA) and Einstein Telescope (ET). In this report, I determine the percentage of binary black hole events detectable by each of three detectors in a year, and detail how the LGWA will give us a full view into binary black hole mergers starting months or even years before their final collisions.

Ever since the gravitational waves were first detected in 2015, the field has advanced at a breakneck pace. The first detections of gravitational waves were made by ground-based observatories such as LIGO, VIRGO, and KAGRA, which observe gravitational waves in the 10-10^3 Hz range. Their designs are well-attuned to the detection of merging stellar-mass binary black holes (BBHs) and merging binary neutron stars (BNS). The future Einstein Telescope (ET) and Cosmic Explorer (CE) will be capable of a much higher resolution and larger frequency range, but will not be able to probe much lower than 1 Hz. Space-based observatories have been proposed to fill this gap, starting with the Laser Interferometer Space Antenna (LISA), which will be able to pick up the signals coming from mergers of black holes of at least 10^6 solar masses at frequency ranges between 10^{-1}-10^{-4} Hz. Millihertz (mHz) detectors also have the advantage of hearing nearby stellar mass BBH mergers from years up to days before the merger, which are then picked up by ground-based detectors as they collide [1]. Going further down the spectrum, pulsar timing arrays such as the North American Nanohertz Gravitational Wave Observatory and the European Pulsar Timing Array are able to detect gravitational waves of frequencies between 10^{-7}-10^{-9}, a range well attuned for picking up signals coming from supermassive black hole binaries [2]. Combining bands to observe phenomena, especially of BBHs, would provide improved measurements of source properties, new constraints on their formation channels, and enable precision tests of general relativity [3].

However, this list of detectors leaves out a crucial part of the gravitational wave spectrum. Several calls for the construction of deciHertz frequency (0.01-1 Hz) will allow for the pursuit of several new scientific goals, such as revealing the formation channels of stellar-mass BBHs, complete the census of BH populations by measuring intermediate-mass black holes (IMBHs), and provide new tests of fundamental physics [4]. Most importantly, it will fill the gap in detection between LISA and ET that will allow for a complete picture of the merger of BBHs, from years before the collision all the way to merger. To facilitate this, astronomers have turned to a rather unorthodox but powerful solution – the Moon [5][4].

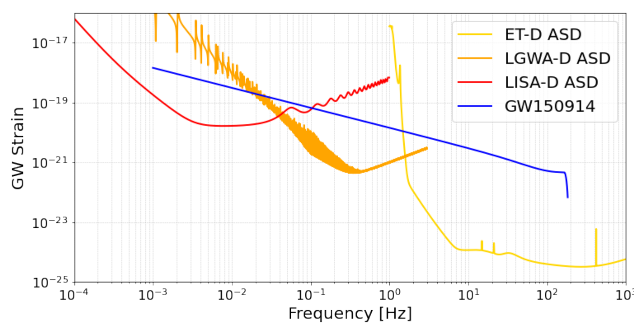

FIG. 1: This graph, with frequency versus strain, depicts GW150914 as it would have been seen by ET, LGWA, and LISA working in tandem. The large area between each of the detector's noise curves and the waves itself is a measure of detectability, called "signal-to-noise ratio" (SNR).

The Lunar Gravitational Wave Antenna (LGWA) is a proposed deciHertz observatory to be deployed during a future Moon mission, composed of an array of high-end seismometers on the Moon to monitor vibrational normal modes of the Moon in the frequency band 0.001 to 1 Hz excited by GWs. The moon is a fantastic candidate for the deployment of a gravitational wave detector because of its characteristics. It lacks an atmosphere, ocean, or major seismic activity that could disrupt detection, as well as having a unique month-long rotation period makes it an ideal place for an observatory. Moonquakes and meteoroid impacts do occur, with several thousand were identified by the Apollo Lunar Surface Experiments Package (ALSEP), but the magnitudes of these events are too small to impact the experiment – the stationary background to the Moonquakes and impacts is so quiet that it was not possible to observe it with ALSEP [5].

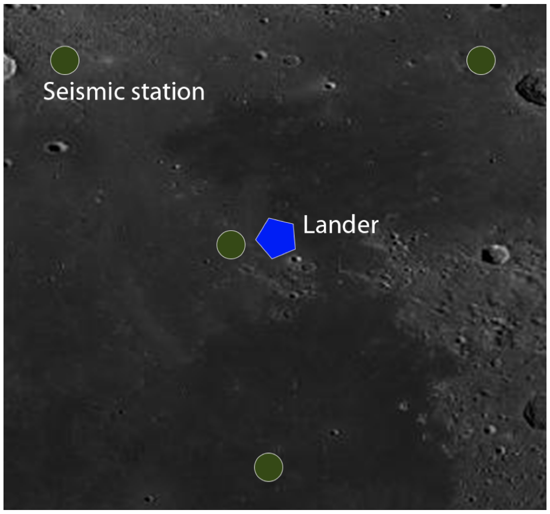

FIG. 2: Depiction of a potential design configuration for the LGWA. Four sets of resonant bars measure the vibrational eigenmodes of the Moon, determining how gravitational waves are making the Moon stretch and squeeze via seismic activity [4].

Using the Moon as a gravitational wave detector has a precedent in history. During the Apollo 17 mission, a set of resonant bar seismometers known as the Lunar Surface Gravimeter was deployed on the Moon. However, a technical failure rendered the data useless, and hopes of a lunar observatory died with the Apollo program [4]. Renewed interest in Moon missions and the confirmed detection of gravitational waves have created a resurgence in calls to put a detector on the Moon. The LGWA has a promising case for being installed in the near future, and will work in tandem with LISA during its lifetime while also serving as a precursor to future deciHertz observatories such as DECIGO and the Big Bang Observer (BBO) [4][6].

The work conducted in this report is two-fold: we both seek to generate an accurate picture of what the BBH population looks like and how useful the LGWA will be as the bridge between the frequency range of ET and LISA for detecting BBH mergers over their total inspiral. This report is divided into three additional major sections. Section I will describe the methods used to generate the population of BBHs as well as the programs used to calculate how detectable a gravitational wave event is. Section II will then discuss, with section III dedicated to conclusions, future work, and acknolwedgements.

I. METHODS

In order to determine the population of BBHs detectable by the three observatories, we first need to generate a population of waveforms to test this on. The model of a population of BBHs is defined by a (source-frame) mass distribution, a spin distribution, and a redshift distribution. These distributions have been constrained by

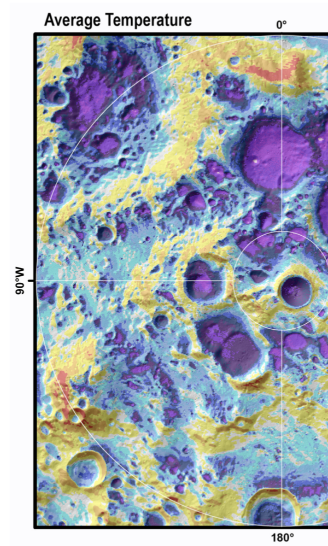

FIG. 3: Average surface temperatures near the south pole of the Moon. The future LGWA will likely have seismic stations inside these polar craters since they are protected from the Sun's heat and do not require a large cryogenic apparatus for cooling [4].

observations of BBHs from LIGO and VIRGO [7]. We use the "power-law + peak" model to describe the masses and the "default spin model" [7], while we use the Madau-Dickinson profile to model the redshift distribution [8]. With the equations described here, we inject several best-fit parameters motivated by BBH observations into our model and generate a larger set of parameters.

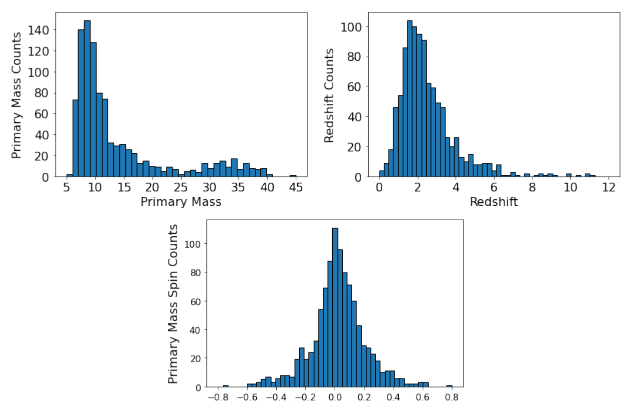

FIG. 4: Graphs of the primary mass, primary spin, and redshift distributions. Mass follows the power-law + peak distribution, redshift follows the Madau-Dickinson profile derived from stars, and spin follows a stranger distribution discussed further down.

The method for generating spin values differs from mass and redshift as we actually generate four parameters: a spin magnitude and the cosine of orientation w.r.t the z axis for both black holes. We also sample several other extrinsic parameters from uniform distributions here, such as sky orientation (θ, ϕ) as well as coalescence and orbit parameters (ι, ψ, GPS time).

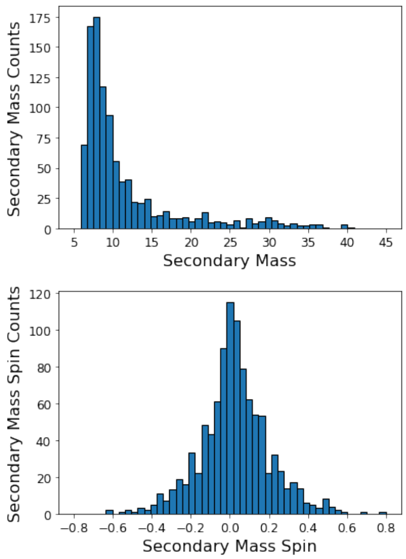

FIG. 5: Graph of the secondary mass and spin distributions.

These model waves are injected into the IMRPhenomD model. It is a full inspiral–merger-ringdown model tuned with NR simulations, which can be used to simulate signals coming from BBH mergers, with non–precessing spins up to $|\chi_z| \ = 0.85$ and mass ratios up to $\frac{m_1}{m_2} < 18$ [9]. This fits very well with the distributions above, which gives more cause to proceed with using it.

Once this is complete, we have to then calculate how detectable a gravitational wave is by an observatory. In general, we assume that the time-domain signal in a GW detector can be written as the combination of the expected signal h_0 and stationary, Gaussian noise n [4].

$$s(t) = h_0(t) + n(t) \tag{1}$$

The noise's statistical properties can be described using the one-sided Power Spectral Density (PSD) $S_n(f)$ defined by the equation below (with tildes for Fourier transforms) [4]:

$$< \tilde{n}^*(f)\tilde{n}(f') >= \frac{1}{2}\delta(f - f')S_n(f) \tag{2}$$

With this, we can then determine an inner product between any two signals $g(t)$ and $h(t)$ [**12**]:

$$(g|h) = 4 * Re \int_0^\infty df \frac{\tilde{g}^*\tilde{h}(f)}{S_n(f)} \tag{3}$$

From this, we can then express the signal-to-noise ratio (SNR) of the true signal with[4]:

$$SNR = (h_0|h_0)^{\frac{1}{2}} \tag{4}$$

The code also accounts for SNRs from multiple detectors by quaring them in quadrature [4]:

$$SNR = \sum \sqrt{SNR_d} \tag{5}$$

This depends on the total area between the noise curve and the wave itself. The higher up the wave is, the more detectable it is.

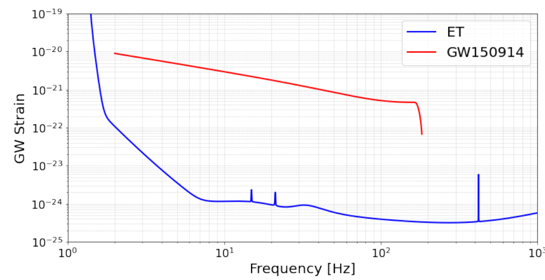

FIG. 6: Here is an example with ET and GW150914, a stellar mass BBH merger event. Due to the low noise curve of the detector and low redshift of the event, we get a very high SNR of 8211.

Calculating the SNR is done with two codes: GWFast and GWFish. Both codes have differences that are important to understand, most notably the injections. GWFish uses sky location (ra and dec, derived from θ and ϕ), luminosity distance, as well as the primary mass and secondary mass, while GWFast utilizes the chirp mass, eta, and a unique variation of the other parameters [10][11]. We used GWFish to calculate the SNR values, and then used GWFast to check that the results of GWFish were accurate. Both codes were shown to align perfectly as shown in the graph above.

With GWFish being accurate, we then used its noise curves for ET, LGWA, and LISA to determine how many BBH signals would be detectable for all three observatories. The calculation gives us how many of the gravitational waves are above the SNR threshhold for each combination of detectors. We chose eight as the SNR threshold as it being a common choice for other projects and motivated by actual detection. Because of its high noise curve, the waves detectable in LISA are the ones that will be detectable over the entire frequency band since all the detectors will record an SNR over eight for those waves. A sample calculation is indicated below, with a population of 1,000 gravitational waves displayed.

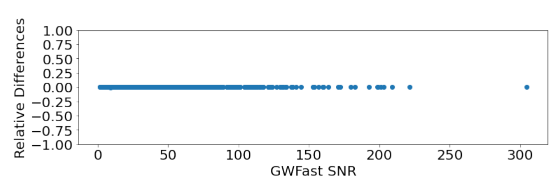

FIG. 7: Graph of the relative differences in SNRs between the two codes.

II. RESULTS

The final calculation was done with 75,000 waves with parameters distributed according to the generation model above. 75,000 is an estimate of how many BBH event signals reach Earth in a year, detectable or not. If the models are accurate, ET will detect a majority of them right before they merge with much detector crossover.

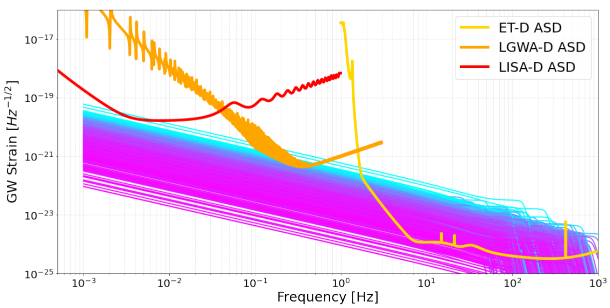

FIG. 8: A trial run for the larger experiment on a population of 1000 GWs. Although ET has a very low noise curve, some waves in the population are still detected by LGWA and LISA, with LGWA detecting more than its counterpart.

From this sample of just 1,000 it is apparent that the LGWA has a slight edge in detection waves over LISA due to its lower noise curve. After running over a population of 75,000 waves, we get the distributions and wave detection frequencies indicated in the graphs and table.

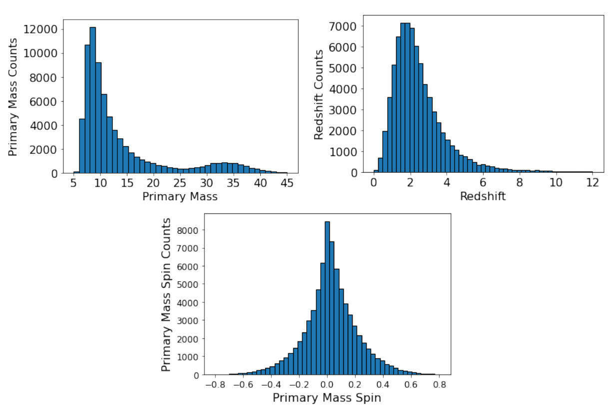

FIG. 9: Graphs of the 75,000-large mass, spin, and redshift distributions.

Detector(s)	# of GWs	%	Max Redshift
ET	66787	89.1	18.396
LISA	2	0.00	0.207
LGWA	93	0.01	1.269

TABLE I: Results of our detectability calculation run over 75,000 simulated gravitational waves. The LGWA appears to detect far more waves than LISA, while ET sees a large share of the population

III. CONCLUSIONS

The case for the LGWA is strong, now bolstered by the fact that serious work can be done toward dissecting BBH populations and analyzing complete mergers from initial detection to final merger. LISA was only able to detect two of these BBH gravitational waves, while the LGWA is able to pull much more weight and give us far more information. Joint detections with LISA will be rare, but not completely ruled out. The LGWA will be capable of detecting far more BBH waves overall in conjunction with ET. As these black holes slowly move toward merger, the LGWA will be able to detect many of these waves before the final inspiral, thus being a crucial player in advancing our understanding of BBHs.

Future expansions of this work will likely involve investigating binary neutron star populations and detectability, as well as other theoretical work involving the elusive intermediate mass BBH merger instead of just stellar mass ones as we studied here. These will require usage of other gravitational wave models designed to account for tidal effects, as well as analyses of other potential distributions. Potential investigations into other BBH distributions based on other models could be warranted as well, creating a well-developed base of knowledge before the LGWA is put up on the Moon.

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