Analyzing Black-Hole Ringdowns with Finite Time Series Inference

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The no-hair theorem states that black holes are characterized entirely by their mass, spin, and charge. When two black holes collide, the remnant black hole rings down to an oblate spheroid, shedding all features on an exponential time scale. Measurements of black-hole ringdowns are necessary for testing general relativity, and gravitational wave data from LIGO/Virgo observations are ideal for this purpose. However, ringdown data is challenging to analyze due to its short duration and location within the signal. Edge effects introduce subtle correlations, which are negligible when analyzing inspiral/merger signals, but detrimental for ringdown analysis. We implement a recently developed method to account for these correlations in order to perform an improved ringdown analysis.

I. INTRODUCTION

According to all known laws of general relativity, a binary black hole (BBH) merger can be characterized by three stages. A BBH signal begins with the inspiral phase where the orbits of the two black holes begin to decay and energy is lost via the emission of gravitational waves (GW); eventually, the black holes merge and then finally ring down to a remnant stage, shedding any other features in the process so that it is only characterized by its mass and spin. For a stellar mass sized black hole, this ringdown phase can happen on the order of a few milliseconds, and any deviations from this expected behavior would violate general relativity [1].

Since commencing observing operations in 2015, Advanced LIGO [2] and VIRGO [3] have detected ∼100 BBH mergers [4], creating an ideal dataset to analyze black-hole ringdowns. However, analyzing black-hole ringdowns proves technically challenging because they are both short in duration and occur near the end of the gravitational-wave signal. It is common to apply a windowing function when analyzing a finite stretch of data to mitigate the Gibbs phenomenon. Specifically, windowing alleviates spectral artefacts that arise as a result of Fourier transforming finite data; these effects become more pronounced the shorter the data segment is. Another problem that arises when analyzing short signals is covariance between frequency bins. The combination of these effects provides a source of systematic error when we analyze GW ringdown signals. In this work we perform parameter estimation on an injected ringdown signal and aim to implement a likelihood with an updated covariance matrix that will improve the accuracy of our analysis.

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The remainder of this paper is organized as follows. In section II, we explain in more detail the challenges in analyzing black-hole ringdowns. In section II, we introduce the methodologies and models that we we will use in our analyses. In section IV we present the results of parameter estimation on the ringdown portion of a GW150914-like injected signal. We provide concluding remarks in V.

II. CHALLENGES OF RINGDOWN ANALYSIS

A. Windowing Gravitational-wave Data

When we analyze gravitational-wave signals, we typically multiply the data by a windowing function to alleviate spectral artefacts caused by the Gibbs phenomenon [5]. This phenomenon occurs whenever time-domain data is Fourier transformed, and in GW data this manifests as a 1/frequency "shelf" or "shoulder" in the amplitude spectral density (ASD) as shown in Figure 1.

Gibbs phenomenon effects worsen the shorter the duration of the data is, which makes ringdown data especially vulnerable to the Gibbs phenomenon and introduces our first challenge. Not only is ringdown data more affected by the Gibbs phenomenon, but the process of windowing may also negatively affect our signal. Since a windowing function essentially sets the noise of our data to zero at the edges while preserving the signal in the middle, a signal's duration may also affect our ability to window. Windowing an especially short signal such as a BBH ringdown runs the risk of "washing out" our data especially if we use windows with non-zero α values (eg. Tukey or Hann windows). This is because the ringdown is placed at the beginning of the piece of signal we are analyzing, and so the tapering effect of a windowing function would set the value of the ring-

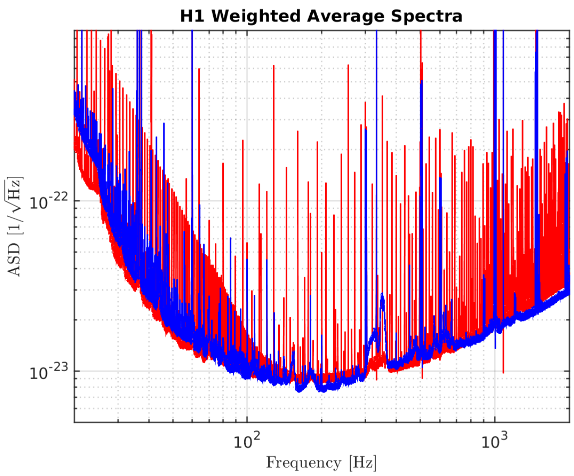

Figure 1. Example of shelfing caused by Gibbs phenomenon in average weighted ASD's of H1 detector during O1 (red) and O2 (blue). [6]

down signal itself to almost zero as shown in Figure 2. Therefore, we plan to eventually apply rectangular windows where α=0 so its tapering nature is not as severe to mitigate this effect.

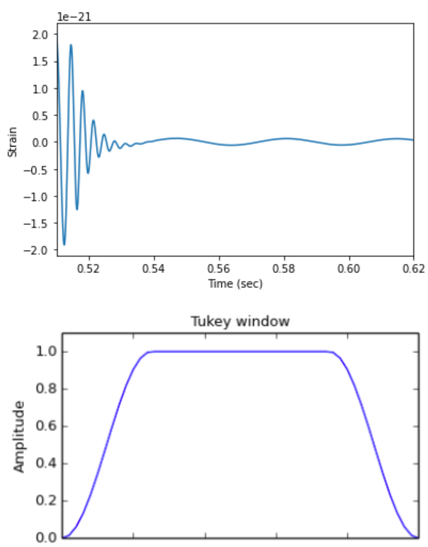

Figure 2. BBH ringdown signal aligned and placed on top of a Tukey window to demonstrate the coinciding location of the ringdown and tapering portion of a window. Multiplying these two would effectively erase the data we are interested in.

B. Frequency Covariance

Fourier transforming truncated data also introduces frequency covariance, which affects the noise model we use in our analysis. To calculate the posterior distributions of our parameters, we generally use the Whittle likelihood function:

$$\mathcal{L}(d|\theta) = \mathcal{N}(\mu = s(\theta), \sigma^2 = \frac{P(f)}{4\Delta f}) \tag{1}$$

where s(θ) is the strain data and P(f) is the power spectral density. This likelihood assumes that the covariance matrix contains only a non-zero diagonal that is characterized by the power spectral density of our signal. However, in real GW signals, frequency bins are not statistically independent and actually have subtle correlations, causing non-zero off-diagonal elements in the covariance matrix. Figure 3 contrasts these two covariance matrices. Just as with the windowing function problem explained previously, frequency covariance becomes more prominent the shorter the signal's duration is, which makes ringdown analysis even more challenging.

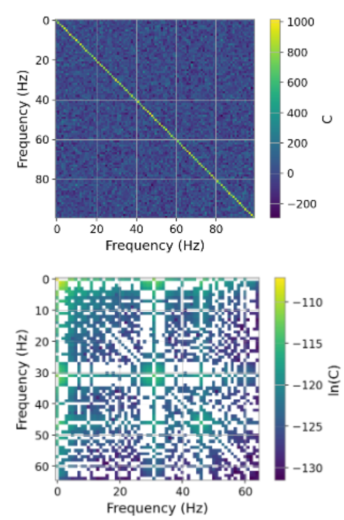

Figure 3. Covariance matrix with only nonzero diagonal element (top) assumed in likelihood versus covariance matrix from .0625 of GW150914 data (bottom) with clear nonzero off-diagonal elements.

This causes inaccuracies for evaluating our posterior distributions, and so we must account for this new covariance matrix in our likelihood which has been developed by Talbot et al. 2021 [7].

III. METHODS

A. Bayesian Inference

We rely on Bayesian inference to perform parameter estimation in this work [8]. Our goal is to calculate the posterior probability distribution of certain parameters–in this case, the parameters of our ringdown signal–which is defined by Bayes' Theorem as:

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{Z}} \tag{2}$$

Here θ represents our source parameters, $\mathcal{L}(d|\theta)$ is the likelihood function, or probability of the detectors measuring data d assuming a model hypothesis, $\pi(\theta)$ is the prior distribution which incorporates any prior knowledge about our parameters, and $\mathcal{Z}$ is the normalization factor, also known as the evidence, which is defined as:

$$\mathcal{Z} = \int \mathcal{L}(d|\theta)\pi(\theta)d\theta \tag{3}$$

For this work we use the software `Bilby`, a Bayesian inference library which infers source properties from individual signals of compact binary coalescences [9].

B. Model of Ringdown Signal

There are two models we must create for our ringdown parameter estimation: the source model which we use to generate our injected waveform, and the recovery model for the parameters we infer. The former inputs a larger segment of time-domain data which is truncated to only the ringdown portion. Because `LALSimulation`, the library we use to generate our waveform [10], causes the signal to wrap around so the ringdown is located at the beginning of the signal, we must truncate the data at the beginning of the time series. For our injections, we use the waveform model `NRSur7dq4`, a surrogate model based on numerical relativity which includes spin-weighted spherical harmonic modes [11]. By taking in the standard 15 extrinsic and intrinsic parameters of a BBH merger signal, the source model returns the cross and plus polarizations of its ringdown signal portion.

Our recovery model for a ringdown signal can be approximated by the damped sinusoid function:

h(t)=$e^{-(t-t_0)/\tau}\sin(2ft\pi + \phi_0)h_0$

where the parameters to be inferred are: t_0, the start time of the ringdown defined in relation to our relative coalescence time; τ, the damping time; f, the fundamental frequency; ϕ_0, the phase; and h_0, the initial amplitude. Figure 4 compares the damped sinusoid model with an actual generated ringdown waveform. Of course, this damped sinusoid model is an approximation because actual ringdown signals are more complex. According to black hole pertubation theory, a ringdown comprises of not just a single damped sinusoid, but rather a superposition of several damped sinusoids with complex frequencies that correspond to different quasinormal modes [12]. Additionally, each ringdown mode may have overtones, which have been explored in previous works such as Isi et al. 2021 [13] and Cotesta et al. 2022 [14]. Complications also arise concerning when a the merger portion of a signal truly ends and the ringdown begins. Although perturbation theory tells us that a GW postmerger signal transitions from a non-linear to a quasi-linear regime, when this moment exactly occurs in time requires more research, which the aforementioned authors have also explored upon.

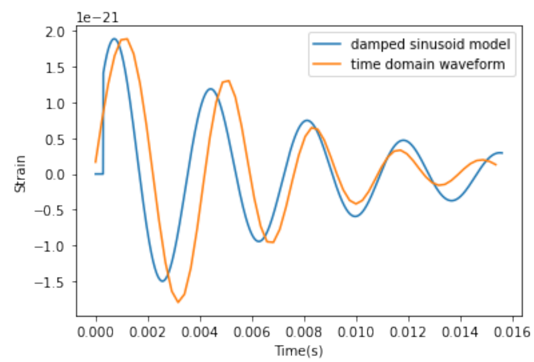

Figure 4. Damped sinusoid model (blue) plotted over actual ringdown waveform (orange) generated using time domain source model.

IV. GW150914 PARAMETER ESTIMATION

Using our damped sinusoid and time domain source models, we are able to perform parameter estimation on a GW150914-like injected signal. We begin by injecting and recovering the same ringdown parameters; as Figure 5 shows, we can see that most of the posterior distributions return close to our expected values; in particular, the amplitude, frequency, and damping time parameters peak at the injected values. However, the t_0 result is surprising as we would expect that parameter to be just as measurable as the others but seems to be returning an uninformative distribution.

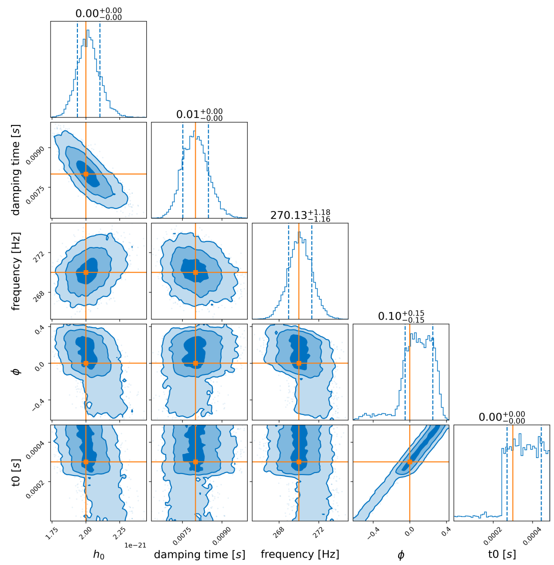

Figure 5. Corner plot of ringdown parameter injection and recovery. Injected values overplotted in orange.

Next, we inject an entire BBH signal and recover only the ringdown parameters as shown in Figure 6. At a glance, this result seems to behave similarly to the previous example. Again, t_0 appears to be unmeasurable and returning the prior distribution this time. Otherwise, the rest of our ringdown parameters seem to be recovered accurately, and we can verify this by calculating what our expected values should be based on the BBH parameters and refer to a table compiled by Berti et al. 2006 [15].

V. CONCLUSIONS

We have created and implemented both a time domain source model to generate a ringdown waveform, as well as a damped sinusoid model which recovers the ringdown parameters from a BBH signal. Using these two models, we managed to recover the ringdown parameters from a GW150914-like injected signal. We recovered these ringdown parameters both by injecting the same ringdown parameters as well as injecting the standard 15 intrinsic and extrinsic BBH parameters. We hope to investigate more into the behavior of the t_0 parameter.

In the future we hope to calculate and implement the new covariance matrix with nonzero off-diagonal elements into our likelihood in order to perform more accurate ringdown parameter estimation. We will ultimately infer ringdown parameters for not just the first quasinormal mode but for others as well and attempt to detect overtones to compare these results with previous findings.

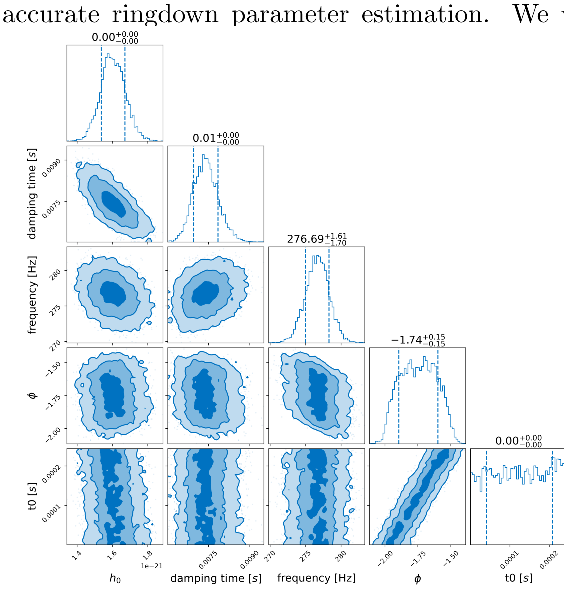

Figure 6. Corner plot of BBH signal injection and ringdown parameter recovery.

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