

Digital Signal Processing Analysis in Readout System for the ALPS-II Heterodyne Detector

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Abstract

The Any Light Particle Search (ALPS) experiment is meant to generate and detect axions and axion-like particles in a wholly laboratory setting. The experiment uses the coherence between two electromagnetic fields, the axion field and the Local Oscillator field. The conversion between an electromagnetic field and an axion-like particle is enabled under an external magnetic field. The current design requires a detection scheme sensitive enough to detect very weak signals with power of the order of 10^{-24}W or 10^{-4} photons/second. We use a heterodyne interferometric detection scheme at the shot noise limit with an I/Q demodulation to process the signal. The focus of this project is the application of techniques to reduce the noise pickups from demodulation and recover a better signal-to-noise ratio through the heterodyne .

Background on axions

While extremely heavy particles are generated by large high-energy accelerators, very light particles at the opposite end of the energy spectrum can help explain important physics phenomena. The ALPS experiment is aimed at exploring particles with masses in the sub-eV range, also called Weakly Interacting Sub-eV Particles (WISPs). The axion is famous among WISP candidates as it is proposed to solve the lack of CP violation in QCD¹.

The axion is also the proposed major candidate of dark matter in the universe since they are weakly interacting and cold axions were naturally produced in the early universe through the vacuum realignment mechanism.

Several experiments are currently searching for WISPs similar to ALPS, such as the OSQAR², which also uses the light-shining-through-walls setup. There is also the CAST and IAXO experiments which source ALPs from the sun³. The ADMX experiment uses a microwave cavity and superconducting magnet to search for axions in the local galactic dark matter halo⁴.

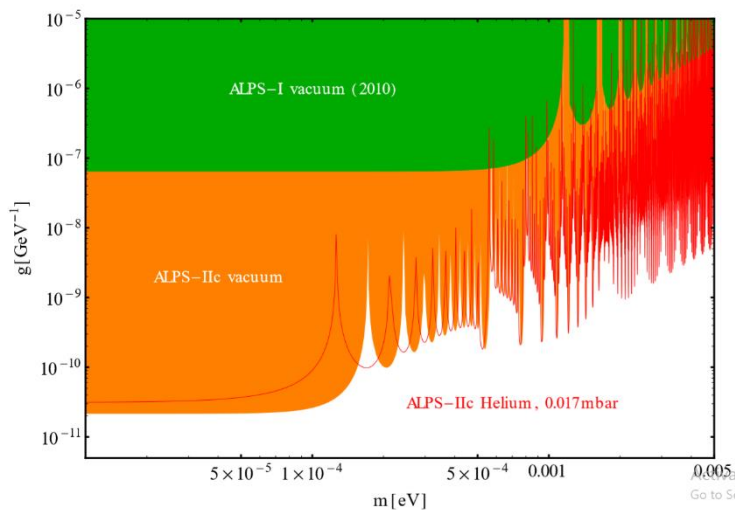


Figure 1. Schematic overview for the prospective sensitivity reach for axion-like particles in the final stage of the ALPS experiment, ALPS-IIc. The upper green shaded area gives the most sensitive bounds on ALPs. In orange is the expected sensitivity reach of ALPS-IIc⁵.

Pierre Sikivie demonstrated in 1983 that it is possible to detect an axion using photon conversion by modifying Maxwell's Equations⁶. Under the presence of a magnetic field, the axion conversion takes place through the "Sikivie effect".

ALPS

The ALPS experiment generates and detects axions and/or ALPs wholly in the laboratory. An external magnetic field allows the conversion between photons to axions and vice versa. Since axions/ALPs do not come from astronomical sources, ALPS is not reliant on cosmological models.

Incident light at a known energy level is injected through a cavity with a strong magnetic field. The injected photons convert into axions or ALPs with the same initial energy by the Primakoff process. Remaining photons will be blocked by wall, while the weakly interacting axions will pass through to the second cavity with a similar magnetic field. Hence the Light-Shining-Through-Wall (LSW) By the Sikivie process, the generated axions are converted back into detectable photons with the same incident energy.

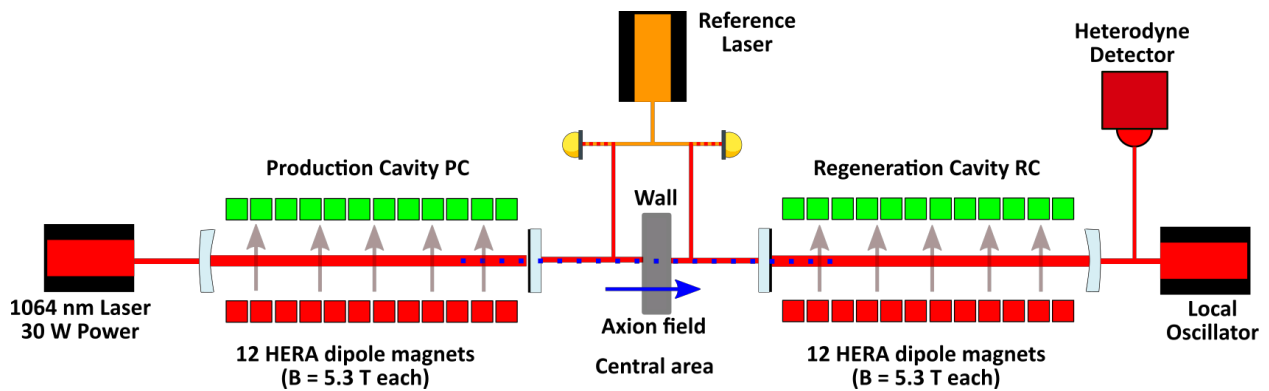


Figure 2. The experimental design for ALPS-II.

Two resonant Fabry-Perot cavities immersed in 5.3T magnetic fields are used to enhance the axion-to-photon conversion⁷. To tune the length of the Production Cavity (RC) on resonance with the injected laser light, Pound-Drever-Hall locking techniques are employed. In the production cavity, a higher circulating power is generated, thus there are more photons to convert into axions. Since energy is conserved, the axions generated will have the same energy as injected photons. The particles cross a light-tight barrier, then enter the Regeneration Cavity (RC).

A local oscillator laser is locked to the RC. Then the PC and RC are connected by looping their phase-locked transmitted fields together with a reference laser. This helps avoid light contamination from the PC side into the RC. This cavity length is locked to the resonance of the injected laser field via error feedback and Pound-Drever-Hall techniques. Regenerated photons in the RC have the same energy and same frequency as the initial beam, thus also resonant in the RC. This resonance allows a power buildup that increases the signal strength of the measured field.

Heterodyne interferometry

Heterodyne interferometry involves overlapping the regenerated signal field with a second reference field, called a local oscillator (LO). We are looking at an electromagnetic wave oscillating at about 280THz, frequencies too high to detect. Thus, we demodulate the signal field with a second field at a nearby frequency:

$$\left| \vec{E}_{\text{signal}} e^{i2\pi f t + \phi_1} + \vec{E}_{\text{LO}} e^{i2\pi(f+f_0)t + \phi_2} \right|^2 = E_{\text{signal}}^2 + E_{\text{LO}}^2 + \kappa E_{\text{signal}} E_{\text{LO}} \cos(2\pi f_0 t + \phi) \quad \text{Eq. 1}$$

where we let $\phi = \phi_2 - \phi_1$.

The combined beam is then incident onto a photodetector (PD) where we measure the power of the beatnote:

$$\left| \sqrt{\bar{P}_{\text{signal}}} e^{i2\pi f t + \phi_1} + \sqrt{\bar{P}_{\text{LO}}} e^{i2\pi(f+f_0)t + \phi_2} \right|^2 = \bar{P}_{\text{signal}} + \bar{P}_{\text{LO}} + 2\sqrt{\bar{P}_{\text{LO}} \bar{P}_{\text{signal}}} \cos(2\pi f_0 t + \phi) \quad \text{Eq. 2}$$

The terms \bar{P}_{signal} and \bar{P}_{LO} lead to DC offsets, while the third term is an AC signal at the difference frequency, f_0 .

For the readout scheme, the PD output is digitized by an analog-to-digital converter (ADC) on-board a Field Programmable Gate Array (FPGA) card. The ADC's digitization rate f_s satisfies the Nyquist criterion to sample signals at f_0 . After a band-pass filter, the signal is split and multiplied by sinusoids off-phase by 90 degrees at a specific demodulation frequency, f_d .

With only signal present, the digitized signal is given by:

$$x_{\text{sig}}[n] = 2G \sqrt{\bar{P}_{\text{LO}} \bar{P}_{\text{signal}}} \cos\left(2\pi \frac{f_{\text{sig}}}{f_s} n + \phi\right) \quad \text{Eq. 3}$$

Once split and split and multiplied into quadratures,

$$\begin{aligned} I[x[n]] &= x[n] \times \cos\left(2\pi \frac{f_d}{f_s} n\right) \\ Q[x[n]] &= x[n] \times \sin\left(2\pi \frac{f_d}{f_s} n\right) \end{aligned} \quad \text{Eq. 4}$$

Letting the amplitude of the beat note $A = 2G(\bar{P}_{\text{LO}} \times \bar{P}_{\text{signal}})^{1/2}$.

Then,

$$\begin{aligned}
 I[x_{\text{sig}}[n]] &= A \cos\left(2\pi \frac{f_{\text{sig}}}{f_s} n + \phi\right) \times \cos\left(2\pi \frac{f_{\text{sig}}}{f_s} n\right) \\
 &= \frac{A}{2} \left[\cos(\phi) + \cos\left(2\pi \frac{2f_{\text{sig}}}{f_s} n + \phi\right) \right] \\
 &= \frac{A}{2} \cos(\phi)
 \end{aligned}
 \tag{Eq. 5}$$

with the $\cos(2\pi n f_{\text{sig}} / f_s)$ removed via filtering, and similarly for the Q quadrature,

$$\begin{aligned}
 Q[x_{\text{sig}}[n]] &= A \cos\left(2\pi \frac{f_{\text{sig}}}{f_s} n + \phi\right) \times \sin\left(2\pi \frac{f_{\text{sig}}}{f_s} n\right) \\
 &= \frac{A}{2} \left[\sin\left(2\pi \frac{2f_{\text{sig}}}{f_s} n + \phi\right) - \sin(\phi) \right] \\
 &= -\frac{A}{2} \sin(\phi)
 \end{aligned}
 \tag{Eq. 6}$$

The quadratures are summed over N samples while assuming that A and ϕ are constant, then used to compute the Z(N) function.

$$\begin{aligned}
 \sum_n^N I[x_{\text{sig}}[n]] &= \sum_n^N \frac{A}{2} \cos(\phi) \\
 &= \frac{AN}{2} \cos(\phi)
 \end{aligned}
 \tag{Eq. 7}$$

and the Q quadrature,

$$\begin{aligned}
 \sum_n^N Q[x_{\text{sig}}[n]] &= \sum_n^N -\frac{A}{2} \sin(\phi) \\
 &= -\frac{AN}{2} \sin(\phi)
 \end{aligned}
 \tag{Eq. 8}$$

It is important to note the factor of 4 lost during the demodulation.

$$\begin{aligned}
 Z(N)_{\text{sig}} &= \frac{\frac{A^2 N^2}{4} \cos^2(\phi) + \frac{A^2 N^2}{4} \sin^2(\phi)}{N^2} \\
 &= \frac{A^2}{4}
 \end{aligned}
 \tag{Eq. 9}$$

We find that Z(N) is directly proportional to the photon rate of the signal field, our quantity of interest, when $A = 2G(\bar{P}_{\text{LO}} \times \bar{P}_{\text{signal}})^{1/2}$ and the demodulation frequency is equal to the signal frequency, as the photon rate of the signal is given by $\bar{P}_{\text{signal}} / h\nu$, where h is planck's constant and ν is the laser frequency.

It is important to note that these calculations are made without considering the noise contributions from other sources in the experiment such as laser intensity noise and electronic noise, for these are below the optical shot noise. For the purposes of this project, the shot noise is simulated using a white Gaussian generator function on MATLAB. Similarly, the Five-Sigma threshold is calculated using the Quantum Efficiency and the time of integration. Furthermore, much of the math in this work is an echo of Zachary Bush's dissertation on Heterodyne Interferometry⁸.

We employ a double demodulation scheme to avoid a DC bias from the FPGA card,

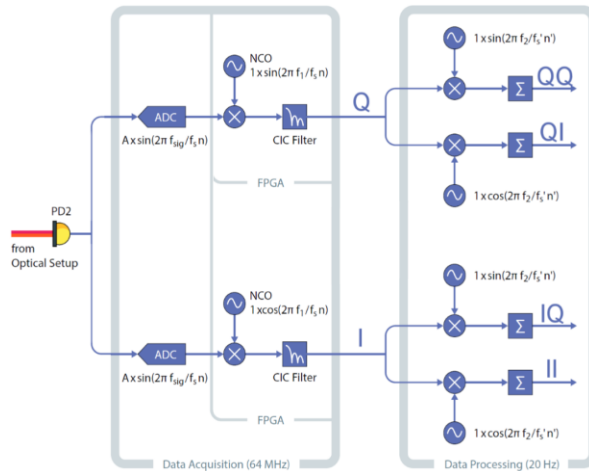


Figure 3. A double demodulation diagram⁸.

A second set of quadratures results from double demodulation,

$$II = \frac{A}{4} + x_{\text{noise}}[n] \cos\left(2\pi \frac{f_1}{f_s} n\right) \cos\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$IQ = x_{\text{noise}}[n] \cos\left(2\pi \frac{f_1}{f_s} n\right) \sin\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$QI = x_{\text{noise}}[n] \sin\left(2\pi \frac{f_1}{f_s} n\right) \cos\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$QQ = -\frac{A}{4} + x_{\text{noise}}[n] \sin\left(2\pi \frac{f_1}{f_s} n\right) \sin\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

Eq. 10

The second loss in signal-to-noise ratio is apparent upon plotting.

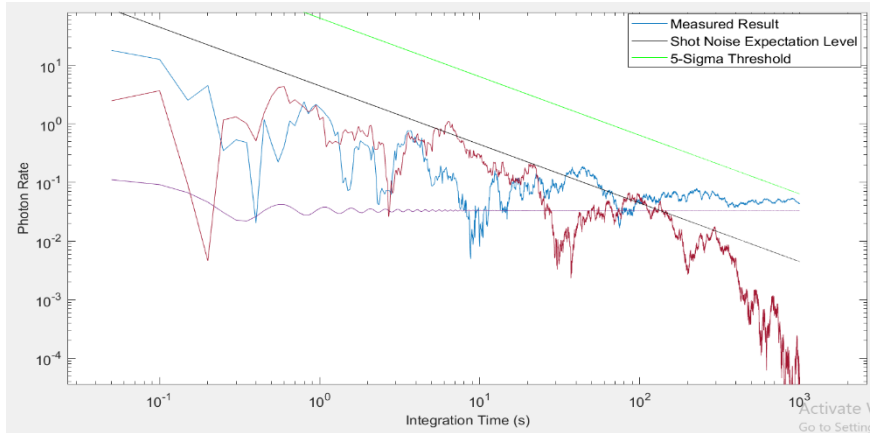


Figure 4. Shows the signal only (purple), the signal inside noise (blue) showing the resulting loss in signal due to the double demodulation, and the noise only (red).

We demodulate once to preserve and reconstruct the signal, but lose a factor of 4. Demodulating again avoids the DC offset, but loses another factor of 4 in the amplitude of the signal. This signal-to-noise ratio loss is apparent, as the signal inside noise does not even cross the 5-Sigma threshold. To remedy this loss in signal, we employ two techniques in the algorithm. Together, they recover the signal-to-noise ratio to original equipment sensitivities.

Linear Combination

After double demodulation, a specific linear combination of the four quadratures can recover a factor of 4 in the signal-to-noise ratio. After double demodulation, we take a specific linear combination,

$$Z_2(N) = \frac{[\sum_{n=1}^N (II - QQ)]^2 + [\sum_{n=1}^N (IQ + QI)]^2}{N^2} \quad \text{Eq. 11}$$

where Z_2 is a new linear combination of the quadratures, and we assume only a beatnote is present at $f_{\text{sig}} = f_1 + f_2$, with $A = 2G(\bar{P}_{\text{LO}} \times \bar{P}_{\text{signal}})^{1/2}$. Solving for the new Z function in terms of the photon rate in the signal field we get

$$\frac{Z_{2,\text{sig}}(N)}{G^2 \bar{P}_{\text{LO}} h\nu} = \frac{\bar{P}_{\text{signal}}}{h\nu} \quad \text{Eq. 12}$$

Taking the same approach to solve for the Z function in which only noise is demodulated, we see

$$\frac{Z_{2,\text{noise}}(N)}{G^2 \bar{P}_{\text{LO}} h\nu} = \frac{2}{\eta\tau} \quad \text{Eq. 13}$$

A reduction in the noise pickup means a gain in signal comparable to a single demodulation stage.

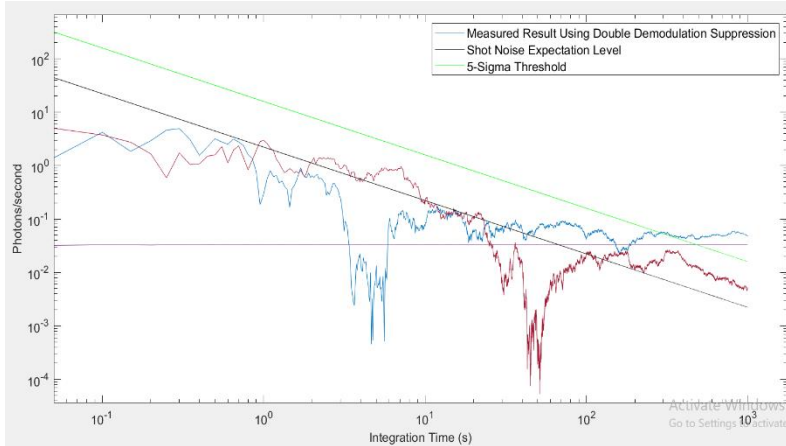


Figure 5. The gain in the signal-to-noise ratio is apparent as the photon signal actually crosses the 5-Sigma threshold.

Phase Search

When we know the phase of the beatnote between the signal field and the LO, we can gain another signal strength factor, and thus recover our original equipment sensitivities before the demodulation stages. The approach for this task is a for-loop algorithm that takes the maximum linear combination value and spits out its respective phase.

$$II = \frac{A}{4} + x_{\text{noise}}[n] \cos\left(2\pi \frac{f_1}{f_s} n\right) \cos\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$IQ = x_{\text{noise}}[n] \cos\left(2\pi \frac{f_1}{f_s} n\right) \sin\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$QI = x_{\text{noise}}[n] \sin\left(2\pi \frac{f_1}{f_s} n\right) \cos\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

$$QQ = -\frac{A}{4} + x_{\text{noise}}[n] \sin\left(2\pi \frac{f_1}{f_s} n\right) \sin\left(2\pi \frac{f_2}{f_s} n + \phi\right)$$

Eq. 14

When the phase is known, the IQ and QI quadratures contain only noise, thus can be ignored. Again taking the specific linear combination, we get

$$Z_2(N) = \frac{[\sum_{n=1}^N (II - QQ)]^2 + [\sum_{n=1}^N (IQ + QI)]^2}{N^2}$$

Eq. 15

Taking the Z function in terms of the signal field we find,

$$\frac{Z_{2,\text{sig}}(N)}{G^2 \bar{P}_{\text{LO}} h\nu} = \frac{\bar{P}_{\text{signal}}}{h\nu}$$

Eq. 16

Calculating the noise term gives

$$\frac{\mathcal{E}[Z_{2,\text{noise}}(N)]}{G^2 \bar{P}_{\text{LO}} h\nu} = \frac{1}{\eta\tau} \quad \text{Eq. 17}$$

Combined, these techniques recover the signal strength lost due to double demodulation, but the phase search technique assumes that there is a signal, that axions do indeed exist and have made it past the wall. Despite the finesse of this phase search algorithm, it would mean that even if there is no axion signal present, it would still demodulate and present a useful Z function. Therefore, to avoid an imposter signal, we require a more robust technique that still finds us the phase between the LO and signal field.

Phase Sweep

Instead of the phase search algorithm, the phase sweep code increments the phi value by five degrees beginning from 0°. The phase that produces the Z function value leading to an output closest to our input photon rate is the correct phase.

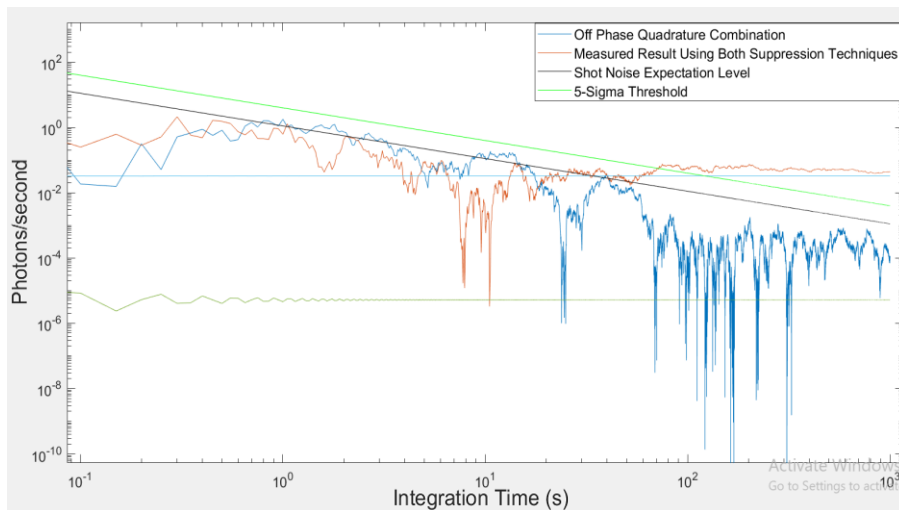


Figure 6. The phase sweep greatly improves the integration time it takes for the signal to cross the 5-Sigma threshold. Also shown are the off-phase quadrature combination made up of QI and IQ. If indeed we have found the correct phase, then the off-phase quadrature combination should be 90 degrees off-phase from our signal, and thus it looks like pure noise. The straight horizontal lines are to reference the pure signal and its off-phase signal demodulated without any noise sources.

As a verification that we have found the correct phase, we plot the photon rate values versus the phase sweep beginning at 0° and incrementing by 5° .

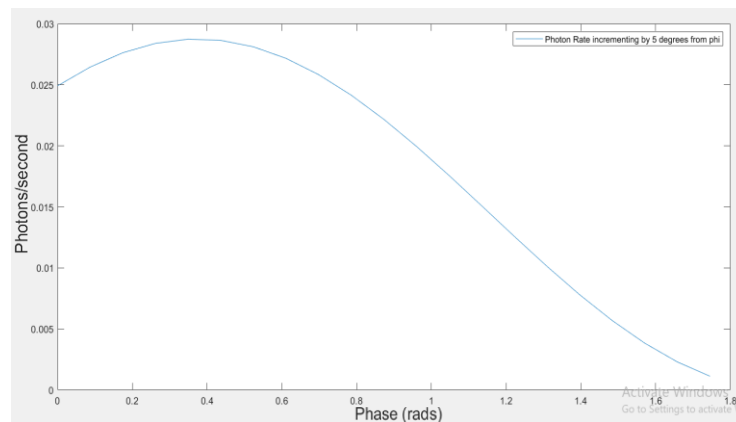


Figure 7. The photon rate values drop as the demodulation deviates from the correct phase.

This algorithm also gives us more control over the phases and the respective photon rate values they produce. We retain each phase, as opposed to the phase search technique, in which only the correct phase value is saved in the code.

Summary

The main part of the experiment is passing a laser through a magnetic field to convert photons into axions, then past a wall into another magnetic field where axions are reconverted into detectable photons. My part in this project is to take the detected and digitized photon signal, write code on MATLAB to demodulate it a second time, then use the linear combination and phase sweep techniques to recover the signal strength lost during both demodulations. The code was written by simulating the parameters and the photodetector signal, as well as part of the data acquisition stage of the experiment. From here, work must be done to figure out the effects of stitching the data together in the case that the signal lock is lost during the experimental run.

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