Inferring the binary black hole redshift distribution

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Abstract

In the past years LIGO-VIRGO have detected five binary black hole mergers; in the universe, there are one hundred thousand binary black hole mergers a year which creates motivation to investigate populations of black holes. New searches are currently being designed to detect the signature of the gravitational wave background of all the distant binary black hole mergers in the universe. This paper will describe the process of how statistical inference can be used to describe the astrophysical parameters of this background. The focus will be in inferring the redshift distribution of the population of black holes which has implications in star formation and primordial black holes.

I. INTRODUCTION

A. 100,000 BBH mergers every year

Black hole binaries, along with other dense bodied binaries, are responsible for the gravitational waves that have been detected by LIGO and Virgo. Since 2015, the year in which the first gravitational wave (GW) was detected, there have been four additionally confirmed gravitational waves from binary black hole (BBH) mergers and one from a binary neutron star (BNS) merger [9]. The current operational interferometers are only able to observe the closest and loudest sources that exist in the local universe, the five BBH mergers were all approximated to be at a redshift on the order of 10^{-1} [1, 2, 6–8]. Now that gravitational waves can be detected and analyzed, this is only the beginning for GW science.

B. Black hole populations

To date, the current BBH merger rate is one merger every 200 seconds corresponding to approximately 150,000 mergers every year [4]. Discussion on this amount of binaries should lead to discussion about black hole populations. Black holes are known to be formed from massive star collapse however, in the early universe primordial black holes may have been formed in regions of highly dense gases. LIGO-Virgo have data on five confirmed BBH mergers meaning there is GW information on the local merger rate but none for any redshift greater than z = 0.3, where information about primordial black holes may exist [12].

FIG. 1 shows an estimate as to what the distribution of star formation might look like as a function of redshift [11]. This distribution is based on mass densities at different redshifts observed using far-UV and far-infrared wavelengths. If black holes are solely formed from star collapse then it can be assumed that a black hole redshift distribution might follow the star formation redshift distribution [14]. However, if primordial black holes were formed then at redshifts greater than 3, the BH redshift distribution might deviate from the star formation redshift distribution.

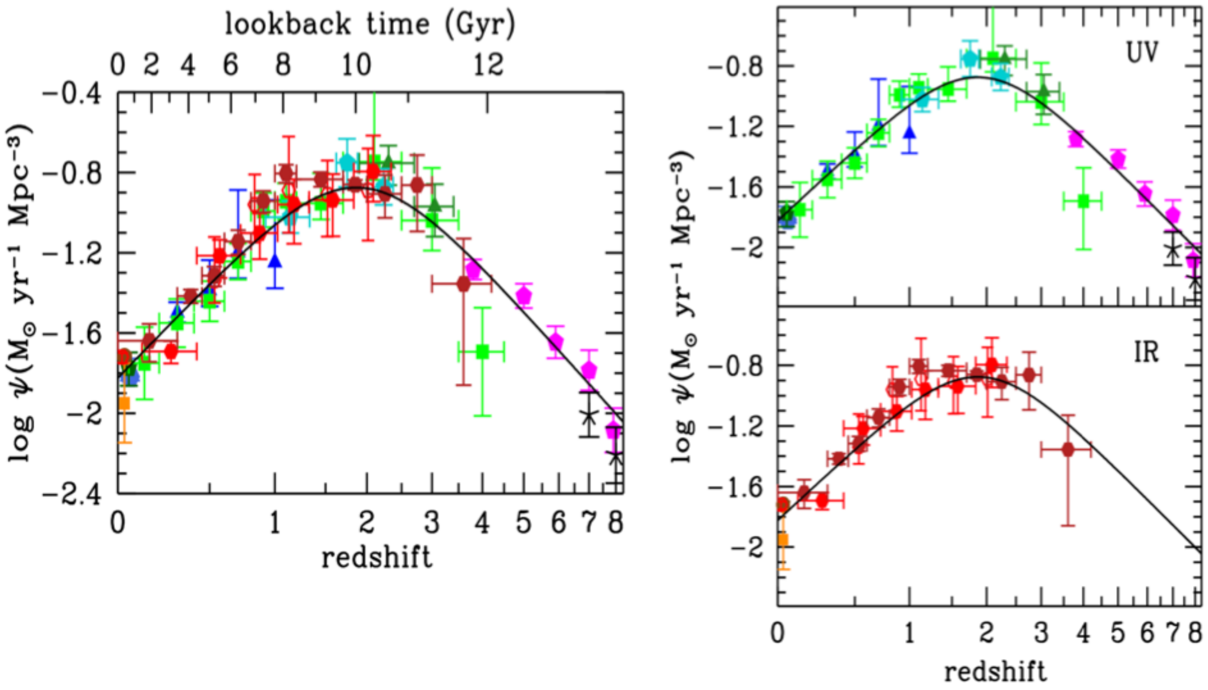

FIG. 1. Madau and Dickinson (2014)

C. BBH gravitational wave background

Once current and future GW detectors reach a sensitivity at which signals can be detected from BBH mergers every 200 seconds, real data can be used to analyze BH populations [3]. Most of the GWs produced from these mergers will be very quiet and contribute to a GW stochastic background. In order to analyze BH populations, loud signals like GW150914 must be considered equally to the quiet signals part of the GW background. Rory Smith and Eric Thrane have devised a new Bayesian search that helps in studying this background [12].

It takes one year's worth of LIGO data and divides it into four-second segments that may be long enough to contain a BBH signal but no evident BNS signal. Each segment is analyzed to determine if there is a signal present or not. FIG. 2 shows how if 5% of the data contains a signal, the search can predict that 5% of the data does contain a signal. Since this is searching through the quiet GW background it is not expected to know which segments contain a signal but rather estimate what percentage of the data does contain a signal.

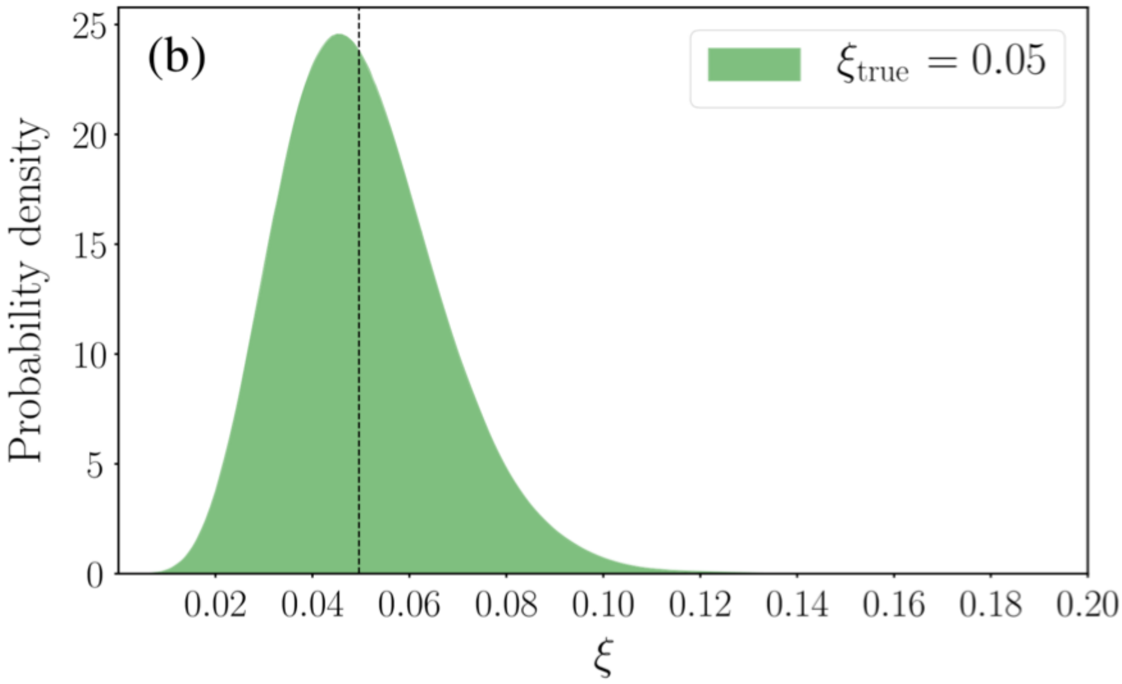

FIG. 2. Smith and Thrane (2018)

II. MODEL

A. Working with simulated data

This project attempts to use this Bayesian framework to analyze the data that does have a signal to infer the properties of the background. Analyzing the background leads questions about whether single event redshift distributions can be used to describe the redshift distribution for the whole black hole population. To attempt this, a model must first be designed to describe the redshift distribution of these black holes. As FIG. 3 describes, this model will be used to generate signals that will in the end follow the model that was created.

B. Broken power law model

The signals will be generated and described by 15 parameters, each of which is extracted from corresponding distributions. Masses for these binaries follow a uniform distribution constrained by LIGO-Virgo's ability to observe the gravitational wave. The masses range

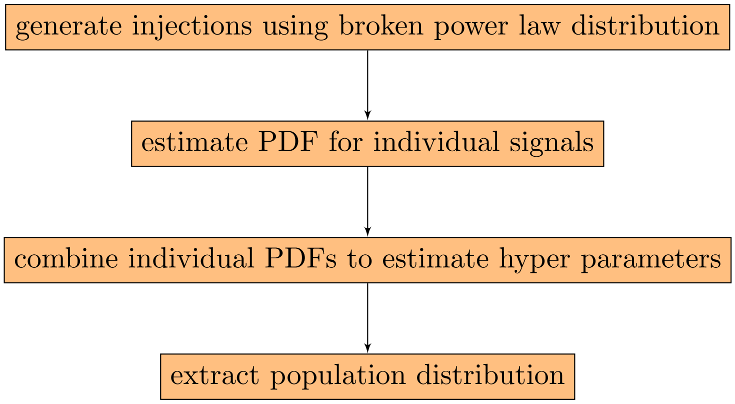

FIG. 3. Pipeline

from 13 to 45 solar masses, the signals produced from binaries of these masses have a duration of less than four seconds at frequencies accessible to LIGO-Virgo's design sensitivity [5]. A model is constructed to describe the distribution for the luminosity distance that is based on the Dickinson-Madau star formation redshift distribution seen in FIG. 1. As a simplified version of the Dickinson-Madau distribution, the luminosity distance probability distribution is described as

$$P(D_L) = f(D_L)\frac{1}{N} \tag{1}$$

where $f(D_L)$ is the broken power law

$$f(D_L) = \begin{cases} (\frac{D_L}{D^T})^{\alpha} & D_L \leq D^T \\ \\ (\frac{D_L}{D^T})^{\beta} & D_L \geq D^T \end{cases}, \tag{2}$$

and N is the normalization factor, which is introduced to ensure the probability is normalized to 1

$$N = \frac{1}{\alpha+1}\left(D^T - \frac{D_{min}^{(\alpha+1)}}{(D^T)^{\alpha}}\right) + \frac{1}{\beta+1}\left(\frac{D_{max}^{(\beta+1)}}{(D^T)^{\beta}} - D^T\right). \tag{3}$$

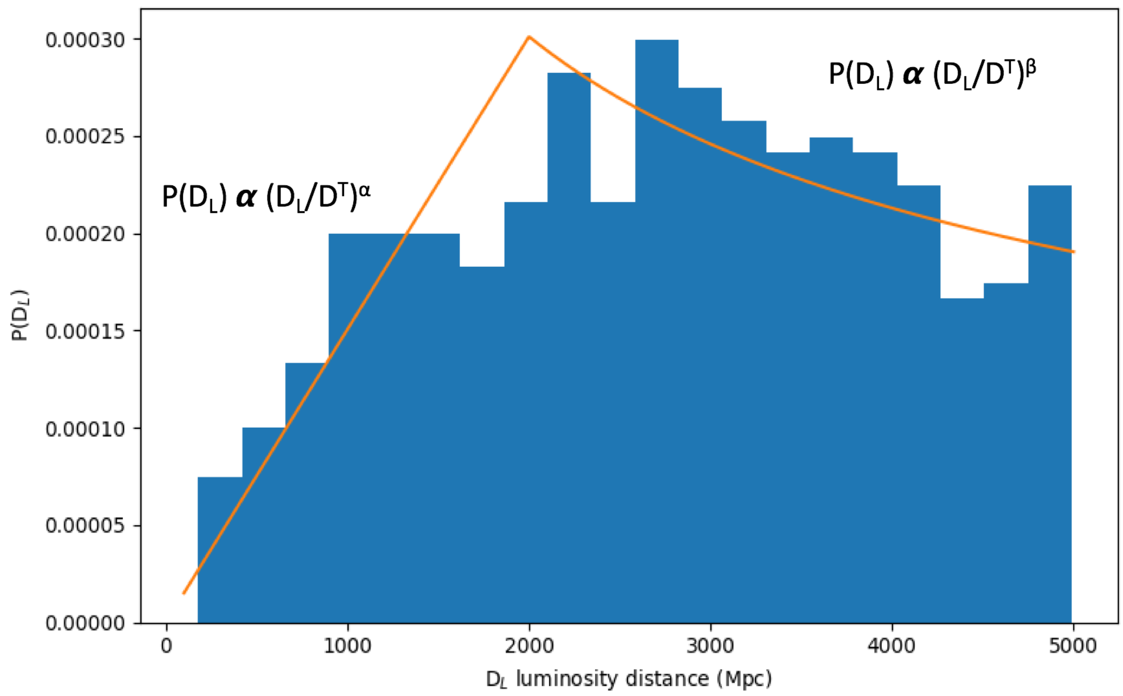

FIG. 4. Produced using TUPAK (Ashton, et al. 2018)

The distribution has cutoffs at $D_{min} = 100$ Mpc and $D_{max} = 5000$ Mpc. The values of α and β are chosen to be 1 and -0.5 respectively, with the turnover value at $D^T = 2000$ Mpc. FIG. 4 shows 500 generated luminosity distances that follow the broken power law model, shown in orange, with the above given values for α, β and D^T. 500 signals correspond to about two months' worth of data if LIGO-Virgo observe a BBH merger every 200 seconds.

C. Parameter Estimation

This project will use the Bayesian framework, discussed previously, to help in estimating parameters at each stage. To first determine the accuracy of this project the most ideal case will be carried out in which only the four-second segments that contain a signal will be gathered. For each segment all fifteen parameters will be estimated. With real data, analyzing the parameters for each event can help in inferring the type of binaries that exist in different regions of the universe.

As FIG. 5 depicts, these parameters are described by distributions. Each parameter of this binary is estimated to be within a range of values. If these were instead exact values, this project would not be necessary. For example, if in the universe there only

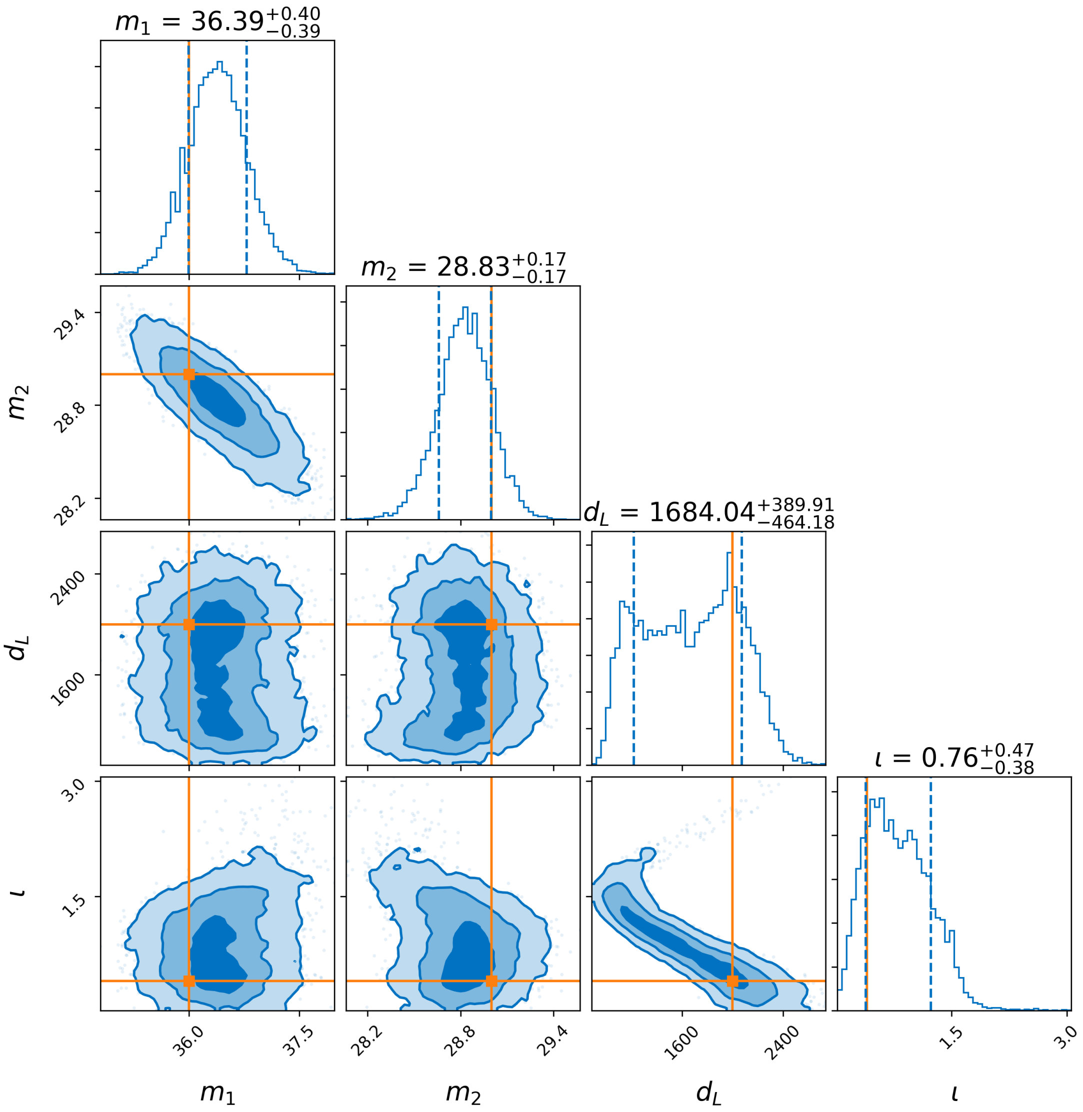

FIG. 5. Produced using TUPAK (Ashton, et al. 2018)

existed a population of 10,000 binaries that all produced a perfectly observable signal, then the parameters for each would be known. Each value, say for mass, could be added to a histogram and that would be a perfect description for the mass population for black holes as the 10,000 binaries would be known explicitly.

However, this is not something that can be done using the distributions for the parameters, so the question is how to combine the individual distributions for all events to describe the population of black holes. After the signals are generated then each will be analyzed using The User friendly Parameter estimAtion Kode (TUPAK), to estimate the marginal distribution for each generated signal [10]. TUPAK uses Bayes theorem to inform about the parameters,

$$P(\,\vec{\theta}\,|\,d) \;\propto\; \mathcal{L}(\,d\,|\,\vec{\theta}\,)\,\Pi\,(\vec{\theta}) \tag{4}$$

where $P(\vec{\theta}|d)$ is the posterior probability distribution function (PDF), $\vec{\theta}$ are the parameters of interest and d is the data presented. $\mathcal{L}(d|\vec{\theta})$ represents the likelihood of the data containing a signal, given a specific set of parameters, which can be better described in Talbot and Thrane (2018) [13]. The prior distribution for this project encodes the astrophysical information of these binaries, as it is the broken power law model detailed in II B, that the injected luminosity distances will follow.

III. RESULTS

A. Hyper PE (preliminary results)

For each of the N events, 500 injections, several sets of parameters were estimated to produce this signal, then as the pipeline for FIG. 3 shows, once the probability distribution function is computed for each signal, then all the distributions for each parameter are combined. This is done using hyper-parameter estimation, where the hyper parameters, Λ, are derived from the prior distribution, which are α, β, and D^T from the broken power law model. TUPAK computes the estimation of these hyper parameters to produce a corner plot that gives their marginal distribution. The likelihood used in this hyper-parameter estimation is given in Talbot and Thrane as well,

$$\mathcal{L}(\,d\,|\,\Lambda\,) \propto \prod_{i}^{N}\sum_{j}^{n_i}\frac{\Pi(\vec{\theta}|\Lambda)}{\Pi(\vec{\theta}|\mathrm{TUPAK})} \tag{5}$$

the denominator is the prior PDF for a single event and the numerator represents the probability of a binary having $\vec{\theta}$ in the broken power law model.

Below, FIG. 6 depicts the estimated values for α, β, and D^T. TUPAK is expected to estimate these hyper parameters so that their real values are within the 90% credible region, with the best estimation being the distributions peak near the true values. As the figure shows, this is not the case just yet.

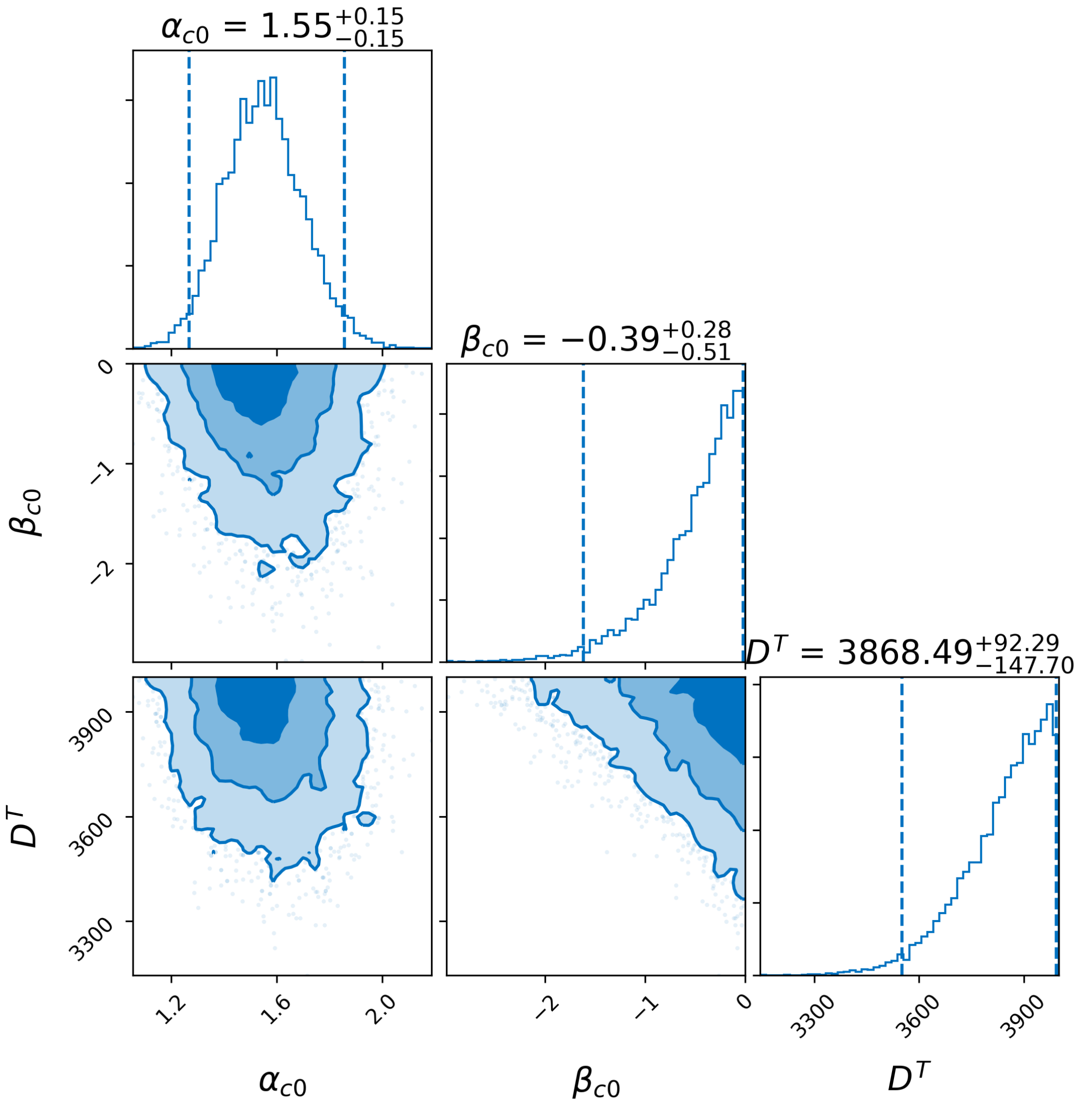

FIG. 6. Produced using TUPAK (Ashton, et al. 2018)

IV. SUMMARY

Although the desired results were not attained, a pipeline, FIG. 3, gives the steps for generating a population of binaries following a specific model, estimating the parameters from the generated signals, and performing hyper-parameter estimation to inform about the population redshift. Once TUPAK can estimate the hyper-parameters with better certainty this work can be extend to analyze two months of simulated data, including segments that contain pure Gaussian noise, rather than only choosing segments containing a signal. A higher injection count would also help in the bias that currently occurs. In the future when detectors reach the necessary sensitivity, this framework and pipeline can be used to answer astrophysically motivated questions about the black holes that exist in the local universe and black holes that exist far and away.

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