Higher Order Mode Matching in Optical Resonators

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The imperfect coupliing flaws between resonant fundamental modes in optical cavities in most modern gravitational wave interferometers such as the one used in LIGO is one source of light loss as some light will mode match with higher order modes. One solution to fix these losses is implementing a method to measure coupling discrepancies by generating modulations onto a laser beam representing a mode mismatch and sending the laser into a optical resonator to measure the beat between our created mode mismatch and a pre-existing mismatch through the reflected beam. An electro-optical lens will describe how mismatched we are by describing the waist size and positioning where a thermo optical lens will correct the mismatch. While no data regarding the mode mismatch of Laguerre-Gaussian modes is recorded, the method to introduce modulations through an electro-optical lens was tested and revealed the EOL does introduce modulation onto the beam, but the signal-to-noise ratio is low that any modulations on the laser is hidden away by the noise.

I. INTRODUCTION

Ever since the detection of a gravitational wave back in 2015 by LIGO, many other gravitational wave scientist have accelerated their research in the hopes of detecting more gravitational waves. Gravitational waves are "ripples" in the fabrics of space-time that are usually caused by violent and energetic events in the universe. These events can range from a variety of comsic event such as two black colliding into each other, a binary star system orbiting each other, or a stellar core collapsing in on itself (supernova explostion). These massive events would send out distorted space-time waves traveling at the speed of light away from the epicenter in the same fashion as a rock falling into a pond and generating outgoing radial waves in the water.

The detection of gravitational waves at the LIGO sight by the use of an interferometer has solidified the fact the use of ground-based interferometers are a feesible method to detect gravitational waves. LIGO currently use the Michelson interferometer setup with arm lengths ranging about 4 kilometers to detect gravitational waves. Inside each arm is a Farby-Perot cavity to let the laser light reflect back and forth at each mirror, allowing for the laser light to travel longer distances to allow for the interferometer to become more sensative to gravitational wave detection as described in Figure 1 [1]. However, this does not mean interferometers do not have sources of error for coupling errors seem to be one source for flaws for interferometers in general. Coupling flaws are imperfections in the general setup and the equipment used. Coupling flaws can be instances of the mirrors not possessing 100 percent reflectivity, lenses used might not be the designated focal length claimed, and more. One particular coupling flaw is the mode matching of the higher order modes inside the optical resonator. The explantion for the previous statement will be explained later, but the idea is some of the laser's power goes into a cavity and resonates with a higher-order mode that will appear in the data while the main goal is to focus all the power into the fundamental mode.

One method to amend the coupling flaws is to decrease the mode mismatches present in the optical resonators by introducing modulations in the form of Laguerre-Gaussian modes onto the laser, creating our own mode mismatch and measure the beat between our created mismatch to a pre-existing mismatch. An electro-optical lens will gives us information about the mismatch through revealing where the waist is positioned and how large it is. Then a thermo-optical lens will be used to correct the mismatch. The Pound-Drever Hall method is implemented to lock the cavity at the fundamental mode or TEM_{00}.

GAUSSIAN BEAMS

A gaussian beam is a beam of light where the electric field profile perpendicular to the beam axis can be described by a gaussian function [2]. In our experiment, it is important to understand how a gaussian beam works because the beam will go through a series of optics and equipment that will modify the beam's characteristics to what we desired. Described in Figure 2 is the main characteristics of a gaussian beam and where the peak of the intensity will reside. w_0 is the beam waist, and it describes the part of the laser in which it is the smallest spot size. Z_R is the Rayleigh range and describes a range in which the beam doesn't diverge too much starting from the position of the waist. Θ is the divergence angle and

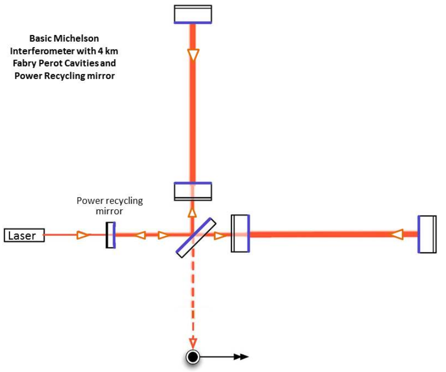


FIG. 1: A Michelson interferometer with Farby-Perot cavities placed inside the arms. The beam splitter splits the into two seperate beams with each traveling down an arm ranging about 4 kilometers. The cavities placed inside allow for the light travel longer distances while also increasing the power inside the cavity, allowing for the interferometer's sensativity and resolution to increase so when a gravitational wave disturbs the light by changing the distance the light travels, the photodiode can detect the change.

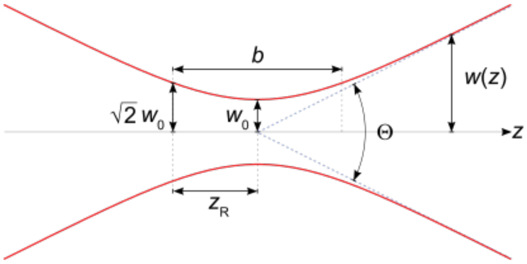


FIG. 2: A picture showing the main characteristics of a gaussian beam. Z_R is unique in that the beam size will always be $\sqrt{2}w_0$ bigger than the beam waist. b is just showing the overall distance between the Rayleigh range since there are two rayleigh ranges for each side of the beam waist.

shows how much the beam diverges best described by the equation

$$\Theta = 2\theta \;= 2\frac{\lambda}{\pi w_0} \tag{1}$$

with λ being the wavelength of the laser. Notice how there is an inverse relation between the beam waist w_0 and divergence angle Θ. This is important because we do not wish to create a beam in which it diverges a lot. The Rayleigh range Z_R is also affected for this for a beam with a large waist will have a long Rayleigh range and vice versa. $w(z)$ describes the beam size at a certain position in the gaussian beam, and the size is describes by,

$$w(z) = w_0\sqrt{1+\frac{z}{z_r}^2}\,. \tag{2}$$

HIGHER ORDER MODES

The gaussian beam is called the fundamental mode or TEM_{00} because it belongs in a family of modes called Hermite-Gaussian (HG) and Laguerre-Gaussian modes. The difference between the these two families of modes is the Hermite Gaussian intensity distributions have a retangular shape that can be described in cartesian coordinates while the Laguerre Gaussian modes are better described in cylindircal coodinates due to their intensity distributions being more circular than rectangular [3]. The gaussian beam is the lowest mode for each of these families. Our focus is more oriented with the formation of Laguerre-Gaussian (LG) modes because we will be introducing these modes in the form of modulations to create our own mode mismatch and compare it an preexisting mode mismatch by comparing the beat betwee the two. Figure 3 shows the intensity distributions of the Laguerre-Gaussian modes as we want to see the TEM_{10} mode when we introduce a modulation onto the laser.

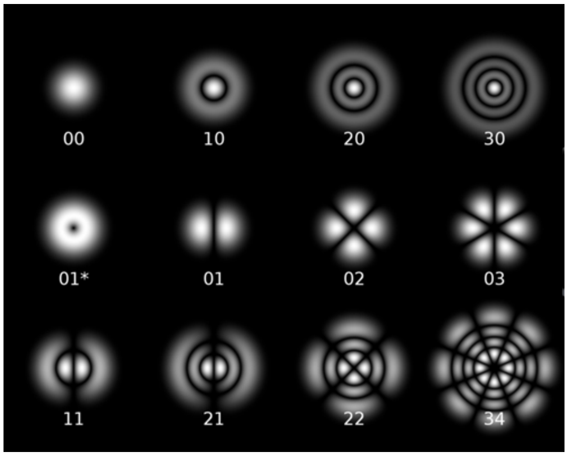


FIG. 3: The Intensity profile of the Laguerre-Gaussian modes. We wish to see the LG_{10} when we apply the modulation with the use of an electro optical lens. Notice also how the Gaussian beam (TEM_{00}) is the lowest mode for the LG modes, which is also the lowest modes for the HG modes [4].

OPTICAL RESONATORS

An optical resonator or cavity is a system in which light at certain frequencies is continuously reflected back and forth without escaping, building up power in the process [5]. There are many ways to construct an optical resonator, but for our purposes, we decided to construct a plano-concave resonator, consisting of a flat mirror where the waist will be located and a sphererical mirror. One important aspect to keep in mind is we wish to construct the cavity in such a way that it has a very high finesse. Finesee tells us about much loss there is in the cavity. One way to increase the finesee is to extend the cavity itself, which is what we did. However, due to the limited spacing, we instead opted out to construct a triangular cavity with very small angles at the top vertex as shown in Figure 10. This will allow us to treat the cavity as two plano-concave cavities with the beam waist inside to be located between the two flat mirrors. It should also be noted the cavity has just enough of an angle at the tip of the triangular cavity where the oncoming and reflected beam at the spherical will not overlap with each other and interfere.

Another aspect about cavities is whether or not they are stable. An optical resonator is considered stable if the beam inside is refocused every time it is reflected off a mirror. If stable, then the beam will not continue to grow and will always refocus while an unstable cavity will continue to enlarge the beam. One way to determine if the cavity is stable is by applying the confinement condition equation

$$0 <= (1 - \frac{d_1}{R_1})(1 - \frac{d_2}{R_2}) <= 1. \tag{3}$$

with d_1 and d_2 being the distances the mirrors are from the beam waist inside the cavity and R_1 and R_2 being the radius of curvature for the mirrors with Figure 4 showing a graphical representation of the confinement condition. This inequality describes if a cavity with certain distances and radius of curvatures will be stable or not. Given that one of the mirrors is a flat mirror, and we can treat our triangular cavity as a linear one with half the parameter through an argument of symmetry, R_1 goes to infinity and d_2 is divided by two, simplifying the confinement condition for our cavity to

$$0 <= (1 - \frac{d_2}{2R_2}) <= 1 \tag{4}$$

where only the spherical mirror's radius of curvature and its placement determines if our cavity is stable.

Cavity resonsance occurs when the laser travels an integer number of wavelengths around the cavity. This is known as the free spectural range (FSR) [5]. FSR is given by the equation,

$$FSR = \frac{c}{2d} \tag{5}$$

with c being the speed of light and d being the length of the cavity. However, we wish to create modulations that are not an integer number of free spectural ranges away. In this case, we would modulate the beam so that it the higher order modes are always an integer number of the same distances away. This is known as Higher-order mode (HOM) spacing, and it describes the distance between higher order modes and is given by the equation,

$$\nu_{m,n,q} = FSR(q + \frac{1+m+n}{\pi} \arccos\sqrt{g_1 g_2}) \tag{6}$$

where m, n represent the travsverse mode indices and q represents the axial mode number. g_1, g_2 represent the the values in the confinement condition $1 - \frac{d_x}{R_x}$. Since we are endeavoring to introduce a modulation that is a distance away from the fundamental mode, m, n, q, go to 0, and $g_1 = 1$ since one of the mirrors is a flat mirror, simplifying the HOM spacing equation to

$$\nu_{0,0,0} = \frac{FSR}{\pi} \arccos\sqrt{g_2}. \tag{7}$$

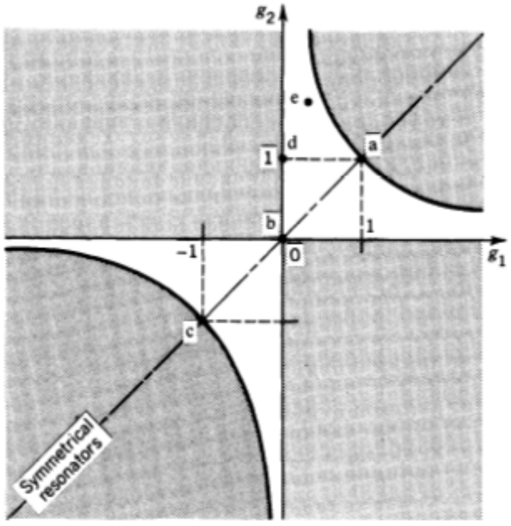


FIG. 4: A graphical representation of the confinement condition inequality. The darkened regions is where the cavity is considered unstable with the labeled letters on the graph representing different optical resonators [5].

POUND-DREVER HALL LOCKING

To mode match a laser to a cavity, both the modes of the laser and the cavity must be the same, and the frequency of the laser must also match the resonate frequency of the cavity. The Pound-Drever-Hall laser locking is a technique that locks the laser frequency to the frequency of the cavity. The way it works is the laser beam is sent through and electro-optical modulator (EOM) that modulates the beam at a specific frequecy determied by an oscillator. The laser is then sent into a cavity where the reflected beam is continually checked by a photodetector. By using the photodector signal and the output the oscillator gives helps to isolate the error signal, which is then goes through a servo amplifier that adjust the laser frequency, locking the laser to the cavity's resonate frequency [6]. Figure 5 gives the basic setup of the Pound-Drever-Hall technique.

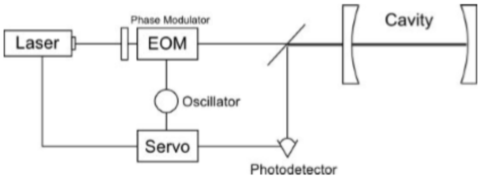


FIG. 5: The setup of the Pound-Drever-Hall laser locking technique [6] The EOM produces modulations dictated by the oscillator while giving an output voltage. The modulated beam then enters a cavity where the reflected beam is monitored by a photodetector. The signal from the reflected beam and the output will help the servo amplifier to isolate the error signal and make adjustments to the laser frequency to match the resonate frequency of the cavity.

THEORY

A laser inside a cavity resonates if the mode and frequency of the laser matches up with the resonant frequency and mode of the cavity. The most common laser mode that will resonate inside the cavity will the TEM_{00} mode. In a perfect setup, all the power in the laser can be found in the TEM_{00}, but since this not a perfect setup, some higher order modes will resonate, taking a small amount of power away from the fundamental as seen in Figure 6. The reason for these higher-order modes resonating is due to coupling flaws that are present like the cavity. Tilts or translations of the cavity may be present that are causing for Hermite-Gaussian modes to appear. Another source for mismatch that causes for Laguerre-Gaussian modes to appear is the waist inside the cavity is not in an optimal position or at the right size, causing for LG modes to appear.

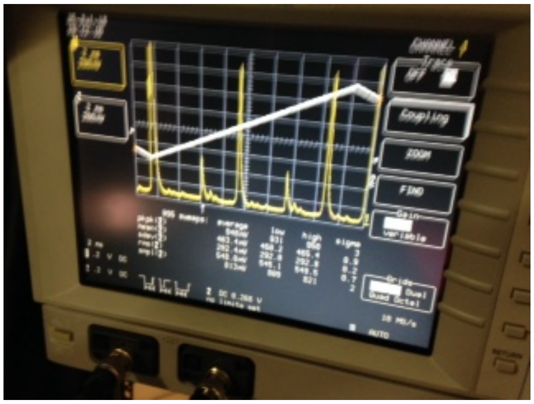


FIG. 6: An oscilliscope reading which modes at certain frequencies are resonating inside the cavity. The large peak represents the fundamental mode while the shorter peaks represent the higher order modes that are also resonating. Taking the area under the curve of all the peaks will tell us about the percentage of power that is going to each resonating mode. In a perfect setup, the fundamental mode is the only mode that should be resonating.

The generation of the LG modes can be described in the following way: consider a gaussian beam with its waist displaced a small distance away b such that $\frac{b}{z_R} = \kappa << 1$,

$$V(r,z) = \sqrt{\frac{1}{\pi}\frac{1}{w(z)}}\, e^{\frac{-r^2}{w^2(z)} - i(\frac{r^2}{2R(z)} - \Phi(z))} \tag{8}$$

where the radius of curvature and the gouy phase shift are

$$R(z) = w_0\sqrt{1+(\frac{z_R}{z})^2}$$
$$\Phi(z) = \arctan(\frac{z}{z_R})$$

and with the LG_{10} mode described as

$$V_1 = \sqrt{\frac{1}{\pi}\frac{1}{w_0}}\, e^{\frac{-r^2}{w_0^2}}(1 - 2\frac{-r^2}{w_0^2}). \tag{9}$$

By taking a Taylor expansion, the beam size, radius of curvature, and gouy phase equations become,

$$w_b = w_0$$
$$R(b) = z_R\kappa^{-1}$$
$$\Phi(b) = \kappa$$

By substituting these equation and taking another Taylor expansion, Equation 8 becomes,

$$V(r,b) = \sqrt{\frac{1}{\pi}\frac{1}{w_0}} e^{\frac{-r^2}{w_0}} (1 - i\kappa(\frac{r^2}{w_0^2} - 1)) \qquad (10)$$

And by doing more simplifications and grouping the higher-order modes, we obtain

$$V(r,b) = (1 + \frac{ib}{2z_R})V_0(r, z = 0) + \frac{ib}{2^{z}_{R}} V_1(r, z = 0). \quad (11)$$

What the mathematical equations are showing is if the waist is not in the right position, the phase of the fundamental changes as well as creating the LG_{10} mode [7]. For more about the mathematical proof, refer to reference 7.

EXPERIMENTAL SET-UP

Before setting up anything, we need to understand the characteristics of our starting beam. Normally, it is specified on the machine generating the laser, but there are other experiments that are taking place with other optical equipment changing the characteristics of the beam, which is described more in Figure 10. A beam profile was taken of the starting laser located at the center of a polarizing beam splitter and noted down several points as shown in Table I.

Distance (mm)	X-waist (mm)	Y-waist (mm)
215.45	.84325	.8882
244,95	.97845	1.026
B274.95	1.1277	1.139
315.95	1.309	1.5054
365.95	1.505	1.573

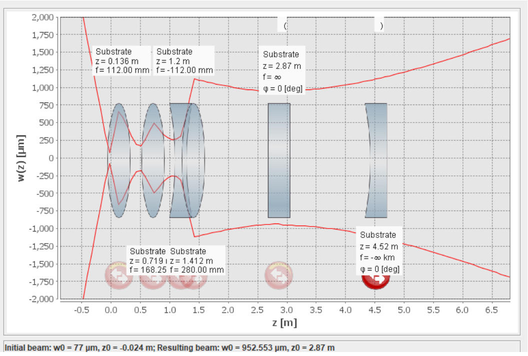
TABLE I: The X and Y waist size at various distances away from the center of the polarizing beam splitter.

Using a polyfit function on Python, we determined that the location of our starting beam waist is $77\mu m$ with its postion 2.4 cm behind the center of the beam splitter.

From there, we needed to match the laser mode to the resonating mode in the cavity with a free spectural range of 95 MHz. However, the beam first needs to obtain a small waist size at certain regions because the EOM and the electro-optical lens (EOL) demand that the beam size to be at a certain size or we risk clipping the beam. For the EOM, the diameter of the entrance into the EOM is 2 millimeters, so we aimed to make the beam size where the EOM should be located less than 300 microns. It is the same situation for the EOL with the only difference being that the beam size has to be 500 microns or less. As for the cavity itself, it has been determined for a mode to be sustained in our triangular cavity, the beam waist has to be 945 microns with the waist located directly in between the two flat mirrors in order to maintain symmetry when reflecting off the mirrors of the cavity as shown in Figure 10.

To assist in determining how to create a setup, we used a program called Just Another Mode Maching Tool (JamMT) to place series of lenses at certain distances to simulate how the beam will behave to create the setup with the established prerequisites. JamMT is a program that uses the ABCD matrix formulation to determine how the beam will transform when going through a lens, calculating the ABCD matrix. As shown in Figure 7, we obtain the desired beam size with regions where the beam size is small enough for it to pass through the EOM at 46cm from the center of the first polarizing beam splitter and EOL at 100cm from the center of the first polarizing beam splitter.

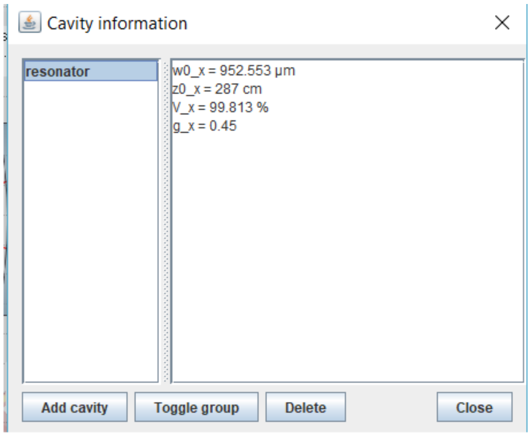
When placing the lenses in the right location and using a beam profiler to understand what is happening, the location of the waists after each len were not in the locatation simulated by JamMT. In order to componsate for this, we placed down the lenses in the locations specified and took the beam profiles after the lenses understand where the waist was located. Going back to JamMT, we modified the lenses' focal lengths to better simulate what was occuring in the actual setup. We found that for each lens, there is about a 12 percent error with the focal lengths specified, causing our waists to form 1 or 2 centimeters away from where JamMT predicted.



Initial beam: w0 = 77 µm, z0 = -0.024 m; Resulting beam: w0 = 952.553 µm, z0 = 2.87 m

FIG. 7: The simulation of the beam after passing through each lens. We obtain the beam sizes needed in order to fit the beam into the EOL and EOM. JamMT is also capable of creating cavities and puttting out values and information about what is occuring in the cavity. As shown, the focal lengths of the lenses have about a 12 percent error in the specified focal length.

However, before adding the cavity in the JamMT simulation, it reported the beam size of 950 microns with its location 3.1 meters away from the starting point, and due to limited spacing, we can only extend the length of the setup 2.87m long, which means that he mode resonating inside the cavity will not be the exact mode require, causing for some mode mismatch. JamMT has another function in that cavities can be inserted into the simulation and obtain information about the constructed cavity. JamMT determined our cavity to have less than 1 percent mode mismatch present as seen in Figure 8. One reason for this is the Rayleigh range is so long (2.61m) the size of the beam does not deviate far from the waist size, having 99 percent mode matching.



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w0_x = 952.553 μm
z0_x = 287 cm
V_x = 99.813 %
g_x = 0.45
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FIG. 8: The simulated parameters of the cavity using JamMT. V_x tells about the mode matching percentage, which only shows less than 1 percent error. g_x tells about the parameter regarding the confinement condition.

Going along with the setup, a beam from a Mephisto 500 NE 1064 nm laser passes through a 100mm lens, half-wave plate and a polarizing beam splitter to create another beam for when another experiment takes place while the lens focuses the beam to create a small enough waist for the EOM to be placed. Another half-wave plate is place in front of the polarizing beam splitter because the next pieces of equipments are the EOM and EOL, and they only function when operating in S-polarization. Once the EOM that introduces are modulation sidebands to lock the cavity is placed down, a 150mm lens is placed down to again to refocus the light so the waist is small enough for the EOL to be placed down. The EOL crystal is made up of Lithium Niobate ($LiNbO_3$). $LiNbO_3$ is a substance that exibits photoelectric effects, which is a phenomenon where its optical properties change when an electric field is introduced [8]. Two specially designed copper plates with wire soldered to them are placed on the top and bottom of the EOL to introduce the electric field as seen in Figure 9 Once our modulations are introduced, the EOL will describe how much mismatch there is by describing the beam waist size and position. The beam then passes through a -100mm lens and 250mm lens mounted on actuators to adjust the positions to increase the mode matching. The beam will then enter the triangular cavity with a base of 15cm and a height of 150cm where a beam will reflect into a photodiode and another will leek through the mirror into another photodiode and a light camerea to see the mode matching.

FIG. 9: An EOL sandwiched between two specially-designed copper electrode plates with wire attatched to a connector to apply a voltage.

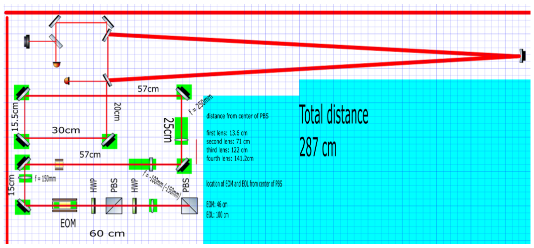

FIG. 10: The schematics of the setup for the experiment with distances and the placement of the lenses, EOL, and EOM with the end point located right in the middle of the base of the triangular cavity. The base is 15cm and the height is 150 cm, making the parameter 315 cm. The laser passes through the EOM to generate modulation sidebands onto the laser to help lock the laser frequency to cavity resonante. The EOL acts as an adjustable lens to describe more about the waist location and size. The laser starts at the polarizing beam splitter because there are other experiments taking place that are irrelevant to the mode matching experiment. This is shown as the blue region where other experiments are taking place and cannot be used for the mode matching experiment.

CONCLUSION

Regretabbly, no data was taken because the setup for the cavity and the configuration to where to place the lenses and other optical equipment took more than two months. When testing out the EOL by passing the beam through it, it had a very minimal effect on the modes present except for the mode appearing due to the polarization. The reason for this is the EOL is producing modulations, but the modulations are so small that the overall noise cloaks the modulations the scan, revealing the signal-to-noise ratio to be too low for any data extraction.

The next step would be to find a way to increase the scanning to obtain a higher signal to noise ratio to see the effects the EOL is having on the laser. After finding a solution to the EOL, a thermo-optical lens will be integrated into the setup so that when we do create our own mode mismatch and obtain information about it through the EOL, the thermo-optical lens can be used to adjust our beam to better align our mismatch to a pre-existing mismatch.

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REFERENCES

[1] L. Caltech, *Ligo's interferometer* (NA).
[2] R. P. Encyclopedia, *Gaussian beams* (NA).
[3] R. Paschotta, *Modes* (2008).
[4] NA, *Higher order modes* (NA).
[5] M. C. T. Bahaa E.A Saleh, *Fundementals of Photonics* (Jonh Wiley and Sons Inc., 1991).
[6] M. Nickerson, *A review of pound-drever-hall laser frequency locking.*
[7] A. Chilton, *Noise in lisa with free space backlink.*
[8] T. A. Maldonado, *Electro-Optic Modulators* (The University of Texas, NA).