Inaccuracies in Correction Parameters and Long Duration Transient Source Recovery

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Binary mergers like GW170817 leave behind remnants which may emit long-duration transient gravitational waves, which evolve over the course of hours or days[2]. Here, we examine preliminary data on the efficacy of "correcting" these rapidly evolving signals such that an existing continuous wave detection method centered on the FrequencyHough may be used to recover the source parameters. This analysis is strictly preliminary and due to time constraints relied upon a very small number of injections for trials. As such, it is necessary to repeat the trials described in this paper with a larger number of injections for statistical considerations. Preliminary results suggest that higher frequency sources and lower spindown sources can be recovered with larger deviations in the initial spindown correction parameter. It is also possible that longer FFTs are more robust for recovering source parameters in the case of inaccurate spindown correction at least for relatively small magnitude errors, while initial amplitude values may play a very limited role in how accurate the correction must be to result in a source recovery.

I. INTRODUCTION

The LIGO Scientific Collaboration and the Virgo Collaboration have successfully detected multiple binary mergers, including the notable GW170817 binary neutron star merger[2]. Much interesting work is being done concerning the signals generated by mergers like these, but also of interest are the gravitational waves that may be generated by post-merger remnants. While it is unlikely that with the sensitivity of the detectors at the time of the merger the form and evolution of the GW170817 remnant can be identified[2], analysis techniques for the kinds of signals that may be produced by these mergers can be developed for potential future detections.

One potential method for investigating these sources utilizes the FrequencyHough, a program designed for the analysis of continuous gravitational waves and used by the Rome group. Because we are interested here in transient waves, using this program requires these more complex, rapidly evolving signals to be corrected for their frequency evolution so that they behave similarly to continuous waves.

In this paper, we examine the robustness of this approach. In particular, we examine how accurate the correction of the frequency evolution must be in order to correctly recover source parameters using the FrequencyHough.

II. NUETRON STARS AND GRAVITATIONAL WAVES

Gravitational waves produced by neutron stars can be formed by several different mechanisms, described by what is known as the braking index, n, of the star, defined in the following way:

$$n = \frac{f\ddot{f}}{\dot{f}^2} \tag{1}$$

where f is the frequency of the neutron star. Because n is an observational quantity, obtaining the braking index of a neutron star gives the frequency evolution over time. This frequency evolution is produced by different physical phenomena within the star. The braking indices we will be discussing are $n = 5$, which describes gravitational waves produced by a rotating neutron star which is nonaxisymmetric (i.e. there is a large surface deformation which is not located along the axis of rotation), and $n = 7$, which describes the frequency evolution of gravitational waves due to changes in the moment of inertia of a neutron star caused by time-varying velocity and density of the fluid which composes the star[1].

If we take the gravitational wave radiation of an isolated neutron star to obey some power law of the following form:

$$\frac{df}{dt} = -k_n f^n \tag{2}$$

where f and n are once again the frequency and spindown of the star and k_n is a positive constant of proportionality, then we can derive the equation to describe the frequency evolution of a neutron star. This frequency evolution is found to be:

$$f(t) = \frac{f_0}{(1 + k_n(n-1)f_0^{n-1}(t-t_0))^{\frac{1}{n-1}}} \tag{3}$$

where f_0 is the frequency at reference time t_0 and t is the observation time on earth. We can further define k_n as the following:

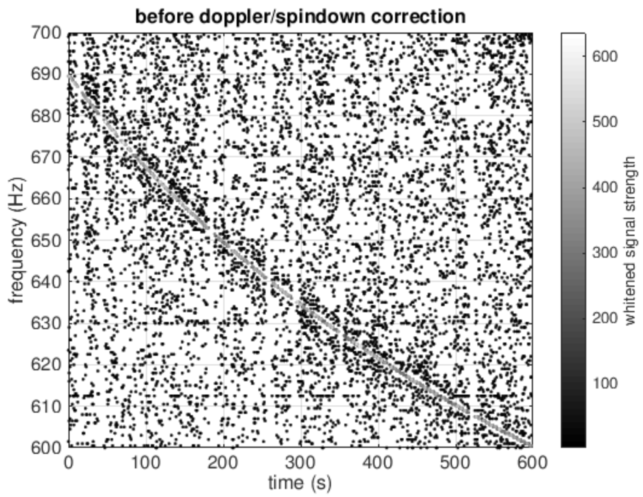


FIG. 1: A 600 second duration peakmap constructed from 2 second FFTs for a source with $f_0 = 690$ Hz, $\dot{f}_0 = -0.25$ Hz/s, and $n = 7$ in white noise. The frequency evolution for this set of source parameters over the 600 seconds is clearly visible and shows the signal decreasing by approximately 90 Hz.

$$k_n = \frac{|\dot{f}_0|}{f_0^n} \tag{4}$$

Continuous gravitational waves are believed to be produced by tiny deformations on the surface of rotating neutron stars. These systems produce gravitational waves with a very small ($O(10^{-12})$) spindowns. These are not perfectly physically described by a braking index, but mathematically speaking they may be described as the case $n = 0$. This case requires careful attention to the magnitude of the spindown to avoid second and third order spindown considerations.

III. THE FREQUENCYHOUGH

The FrequencyHough is a program used by the Rome group to search for continuous gravitational wave signals[1]. Taking data from a series of cleaned 1024 second FFTs (Fast Fourier Transforms), something known as a peakmap is constructed for each point on the sky. A peakmap is a method of representing the frequency of peaks (values which rise above a given threshold and which are also a local maximum) in the data for a given time.

In the case of continuous waves, we can describe the frequency evolution in the following way:

$$f(t) = f_0 + \dot{f}(t - t_0) \tag{5}$$

Rearranging this equation gives the following relationship:

$$\dot{f} = -\frac{f_0}{(t - t_0)} + \frac{f}{(t - t_0)} \tag{6}$$

So now for each data point in the frequency and time domain of the peakmap, the equation of a line in the initial frequency and spindown plane is formed. By observing where these lines most frequently intersect, we can select the likely candidates for the actual parameters of the source in question and do more rigorous analysis on those candidate signals to determine if a gravitational wave signal is truly present.

A generalized version of the FrequencyHough is being developed which utilizes a coordinate transform to analyze these transient signals in a space in which they obey a linear relationship. Specifically, a new coordinate x is utilized and defined to be

$$x = \frac{1}{f^{n-1}}; x_0 = \frac{1}{f_0^{n-1}} \tag{7}$$

This new parameter x_0 and the previously defined k_n, which obey Equation 8, are the parameters which the FrequencyHough attempts to identify for an input peakmap constructed on a $(t - t_0, x)$ plane[1]. These parameters, once identified, can be translated into the initial spindown and frequency. This is one possible method of selecting candidate source parameters which may then be used to prepare a noncontinuous gravitational wave signal for the original FrequencyHough in a manner which will now be described.

$$k_n = -\frac{x_0}{(n-1)(t - t_0)} + \frac{x}{(n-1)(t - t_0)} \tag{8}$$

In order to use the FrequencyHough for non-continuous gravitational wave signals, which have a nonlinear frequency evolution, we use Equation 3 to monochromatize the signal. In order to do this, the original peakmap must be generated with with an FFT length such that the signal drifts no more than one frequency bin within one FFT, where one bin is defined for an FFT to be T_{FFT}^{-1} Hz. Selecting some T_{FFT} therefore sets the maximum spindown which can be analyzed. This relationship is described with the following equation:

$$|\dot{f}_{max}| = \frac{1}{T_{FFT}^2} \tag{9}$$

Once the correction is made for both the frequency evolution and the Doppler effect, the corrected peakmap we have generated will be similar to those of the continuous wave case, and the original FrequencyHough can be used in the same way as previously described to find the likely initial frequency and *residual* spindown of the peakmap. In the ideal case, we know the parameters of the signal exactly and can therefore make an exact correction,

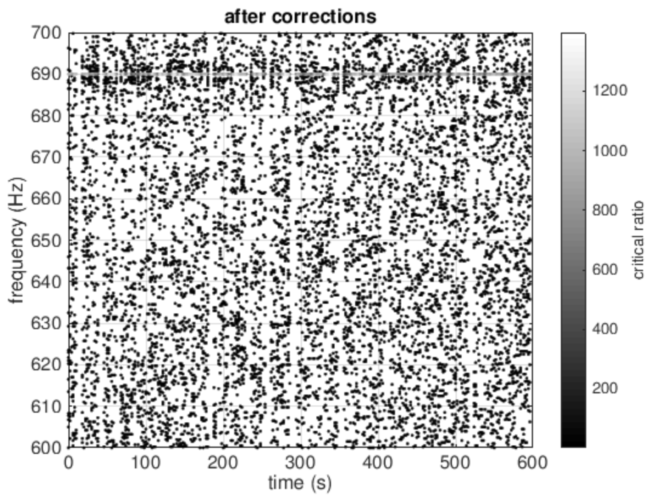


FIG. 2: A 600 second duration peakmap constructed from 2 second FFTs for a source with $f_0 = 690$ Hz, $\dot{f}_0 = -0.25$ Hz/s, and $n = 7$ in white noise. The signal in this case has been Doppler and spindown corrected using Equation 3, so the signal appears monochromatic at its initial frequency of 690 Hz.

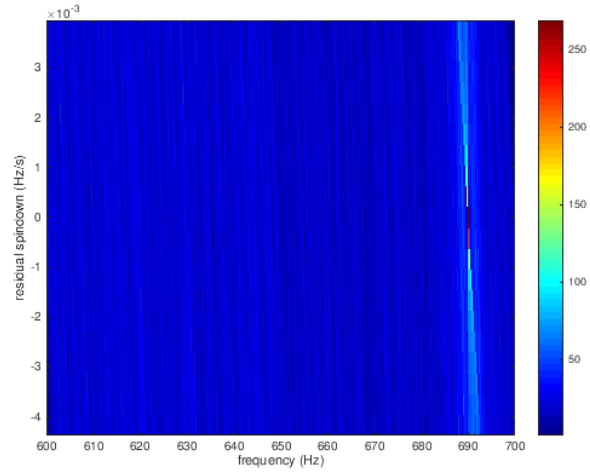


FIG. 3: This figure shows an example of a Hough map, which shows points of intersection of the lines described in III. This plot was generated using corrected 600 second duration peakmap constructed from 2 second FFTs for a source with $f_0 = 690$ Hz, $\dot{f}_0 = -0.25$ Hz/s, and $n = 7$ in white noise.

resulting in a perfectly monochromatic signal. In more realistic scenarios (like the use of the generalized FrequencyHough described in the previous paragraph), we might have parameters which are close to the true source parameters, but which — when used to correct the signal — result in some small residual spindown. This residual spindown will be the parameter the Hough transform would attempt to identify. These candidates for initial frequency and residual spindown, applied to the correction parameters used to monochromatize the signal, give the candidate parameters for the source in question. In the case of injected signals, these candidates can be compared to the injected source parameters to understand how changes in the correction parameters affect source recovery.

IV. RESULTS OF FALSE SIGNAL CORRECTION

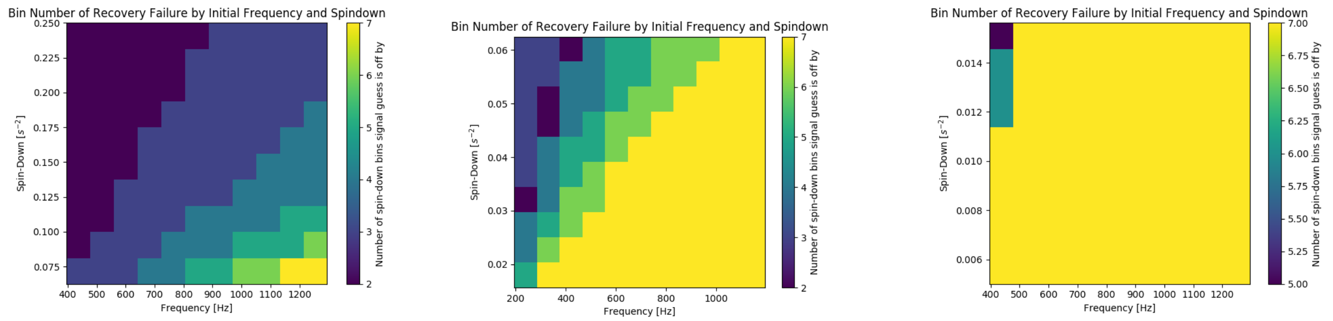
We seek to quantify how inaccurate our correction of a signal can be while still resulting in a recovery of the source in the follow-up. We will focus specifically on how inaccurate the selection of the initial spindown can be given other fixed parameters. We will do this by injecting signals into real detector noise (in this case from the Livingston detector following the GW170817 merger), correcting for parameters which are the same as the source parameters with the exception of the initial spindown (which will be varied by a known amount), running these peakmaps through the FrequencyHough and selecting candidates, and then identifying if the source parameters were recovered to within three frequency and spindown bins by any of the candidates.

Because we survey a range of initial spindowns including the maximum spindown for a given T_{FFT} in all of these trials, the spindown being shifted by N bins refers to an decrease in spindown magnitude (i.e. a spindown which is less negative), as in a real scenario, spindowns greater than the maximum allowed for an FFT length would be analyzed with a shorter T_{FFT}.

Due to time constraints, these results were produced using only 10 injections per set of parameters, and should be treated as preliminary to future trials. For more on the future of this analysis see Section V.

A. Variations in Initial Frequency and Spindown

A trial was conducted in which the initial frequency and spindown were varied for fixed braking index and initial amplitude (h_0). For each set of parameters, the spindown used in the correction was moved away from the source spin down one bin at a time until the source parameters were no longer recovered as defined previously with a 90% confidence threshold. Figure 4 shows the number of bins away the correction parameters were when the recovery failed as a function of the initial frequency and spindown of the source. One can observe that the maximum spindown for the FFT length (In the case of Figure 4a for example, $T_{FFT} = 2$ s and so $|\dot{f}_{max}| = 0.25$ Hz/s) sets the lower limit for the number of bins the correction could deviate by for all frequencies. It is for this reason that in subsequent analysis we utilize the maximum initial spindown for the T_{FFT} in question, as this represents the parameters which are most sensi-



(a) (b) (c)

FIG. 4: This figure represents three different cases of the same source recovery test. Each figure gives the number of spindown bins by which the correction parameters deviate from the source to such an extent that the source parameters are no longer recovered by the traditional FrequencyHough procedure described in Section III. Figure 4a is the case $T_{FFT} = 2$ s, Figure 4b is the case $T_{FFT} = 4$ s, and Figure 4c is the case $T_{FFT} = 8$ s. All three figures were generated with from a 600 second duration signal with braking index 7 in real Livingston detector noise from shortly after GW170817. The spindown range for each case reflects the range for which the T_{FFT} in question is best suited.

tive to inaccuracies in spindown.

Figure 4 shows also that as the initial frequency increases and the initial spindown decreases in magnitude, it is possible to correct the signal with a greater error in initial spindown while still recovering the source. This is perhaps because having a small initial spindown means that the frequency will evolve more slowly and may indicate a better recovery rate for signals at higher frequency. Additionally, it should be noted that for longer FFTs, there is less variation in the number of bins across the frequency and spindown ranges. This may reinforce the the hypothesis that sources with smaller initial spindowns may be less sensitive to errors in parameter correction (as longer FFTs are confined to smaller spindown ranges). It may also suggest that overall signal recovery is more sensitive to errors in initial spindown for shorter-duration FFTs. To elaborate, one frequency bin is defined to be

$$\text{bin}_{sd} = \frac{1}{T_{FFT} T_{tot}} \tag{10}$$

where T_{tot} is the duration of the signal. So, in the case where $T_{FFT} = 2$ s, each bin count represents a change in initial spindown of approximately 8.33×10^{-4} Hz/s, while for the $T_{FFT} = 8$ s case, each bin only amounts to a 2.08×10^{-4} Hz/s change in initial spindown. This would mean that, while a 1.45×10^{-3} Hz/s deviation in initial spindown corresponds to a 7 bin deviation for the case $T_{FFT} = 8$ s (as seen in much of Figure 4c), this only corresponds to a 1.75 bin decrease in spindown magnitude for the $T_{FFT} = 2$ s case. Also, observing Figures 4a and 4c and referring to Equation 10, the correction spindown can deviate in the low-spindown, high-frequency range of the $T_{FFT} = 2$ s case up to 4 times further than in the $T_{FFT} = 8$ s case. However, the threshold which defines a recovery (which in this case is that one of the candidates selected by the FrequencyHough is within three initial frequency and spindown bins in total of the source parameters) is FFT dependent, so while the magnitude of the spindown can deviate by a larger amount for shorter FFTs, the larger bin sizes mean that the candidates need also be less accurate. Further trials are necessary to fully understand the sensitivity relationship between the length of the FFTs and the spindown correction sensitivity, but this preliminary data may suggest that longer FFTs may be more robust for source parameter recovery in that a larger portion of their parameter space can deviate in initial spindown correction by a relatively high number of bins. However, more trials must be conducted to verify that this effect is indeed due to the FFT length and not attributable to the smaller spindown magnitudes. These patterns are consistent with for braking indices 5 and 7.

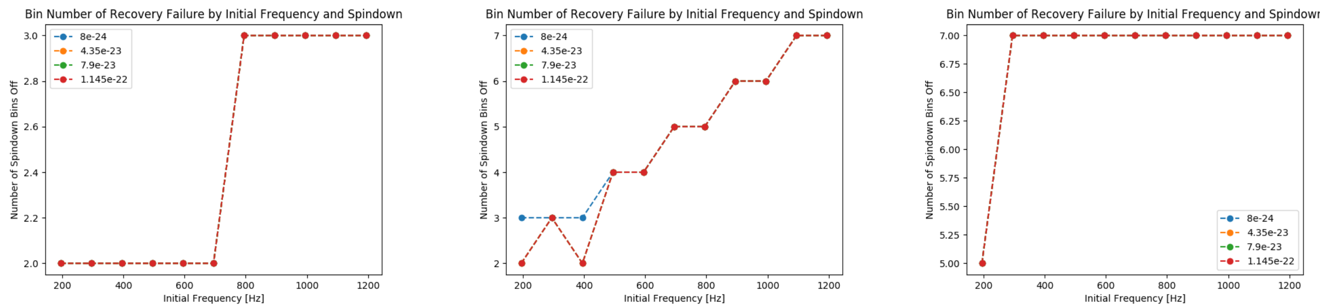
B. Variations in Initial Signal Amplitude

A trial was also conducted in which the initial strength of the signal was varied in the form of the initial amplitude, h_0. The evolution of the signal amplitude with $n = 5$ can be described with the following equation:

$$h(t) = \frac{4\pi^2 G}{c^4} \frac{\epsilon I_{zz}}{d} f(t)^2 \tag{11}$$

where G is Newton's gravitational constant, c is the speed of light, and ϵ, I_{zz}, and $f(t)$ are the ellipticity, moment of inertia, and frequency, respectively, of the source.

For a source with $n = 7$ the amplitude evolution behaves differently and can be modeled instead with this equation:



(a) (b) (c)

FIG. 5: This figure represents three different cases of the same source recovery test. Each figure gives the number of spindown bins by which the correction parameters deviate from the source to such an extent that the source parameters are no longer recovered by the traditional FrequencyHough procedure described in Section III. Figure 5a is the case $T_{FFT} = 2$ s, Figure 5b is the case $T_{FFT} = 4$ s, and Figure 5c is the case $T_{FFT} = 8$ s. All three figures were generated with from a 600 second duration signal with braking index 7 in real detector noise from shortly after GW170817. The spindown for each case is the maximum spindown for the T_{FFT} in question.

$$h(t) = 1.8 \times 10^{-24}(\frac{20\text{Mpc}}{d})(\frac{f(t)}{1000\text{Hz}})^3\alpha \qquad (12)$$

where α is the saturation amplitude of the r-mode that is described by the $n = 7$ case.

For this trial, the braking index and initial spindown of a source was fixed and, for a series of h_0 values, the failure point (described once again by a number of spindown bins by which the correction parameters deviated from the source) for injection recovery was found as a function of initial frequency.

The results of this preliminary trial suggests some variation in the sensitivity of this recovery method for different initial frequencies, but relative consistency across the small range of h_0 values which were tested. The points of inconsistency where the smallest value of h_0 is recovered for additional frequency bins may be a reflection of the small number of injections used in this trial or of some artifact in the noise of the detector. Beyond this, the results of this trial are consistent with the results in Section IV A, with higher initial frequencies appearing to have a greater margin for error in the initial spindown correction parameter. The consistency of the bin deviation threshold between h_0 values suggests that the strength of the signal may not be a significant factor in source recovery if the signal is within the detectable range of amplitudes. However, the errant detection of the weaker signal suggests that more rigorous testing is required for verification of this hypothesis.

V. FUTURE ANALYSIS AND CONCLUSIONS

As previously stated, the results presented in this paper are preliminary and warrant further investigation. These trials used a very small number of injections and thus have great uncertainty attached to their statistics. With a higher number of injections, the dimensions of the recovery parameter space could be much more reliably known.

Additionally, these trials focused only on varying the spindown correction parameter, but it follows from Equation 3 that the signal evolution depends on the initial frequency as well. Understanding how an incorrect initial frequency correction would impact the source recovery and how this error and an incorrect spindown correction might interact in a real follow-up is important to understand when making choices for how a signal analysis pipelines might select correction parameters. Also of interest is the impact of the length of the FFTs which are utilized. Additionally, the impact (or lack thereof) of the signal amplitude is important to explore, particularly because there appears to be a possibility of false recovery as in the case of the weakest amplitude in Figure 5.

How these parameters and their interaction with each other might impact source candidate selection and the follow-up is important to understand if the FrequencyHough analysis process is to be successfully adapted to recover long-duration transient gravitational waves. These preliminary results suggest better recovery for correction parameters which deviate from those of the source at higher initial frequencies and smaller initial spindowns. They additionally suggest a larger margin of error for shorter FFTs but better accuracy for longer FFTs, while seeming to show little deviation for different values of initial amplitude.

[1] Miller, A., et al. A method to search for long duration gravitational waves from isolated neutron stars using the generalized FrequencyHough. In preparation, 2018.
[2] Abbott, B. P., et al. Search for post-merger gravitational waves from the remnant of the binary neutron star merger GW170817 arXiv preprint arXiv:1710.09320 (2017).