Improving Gravitational Wave Detection Statistic using an ensemble of Pulsars

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The purpose of this paper is to present the preliminary results of the Rome group's study on improving the detection efficiency of continuous gravitational waves using an ensemble of pulsars. The study is based off the current targeted search method that looks for CGWs from asymmetrically rotating neutron stars. By combining the detection statistic of individual pulsars using different weights, a new detection statistic is developed, which improves the detection efficiency of CGWs. We combine the detection statistics from 9 randomly generated pulsars for 4 different optimal SNRs and compare them to equivalent noise only distributions.

I. INTRODUCTION

Gravitational waves, first predicted by Einstein's General Theory of Relativity, have been directly detected twice in the past 12 months having eluded detection for nearly a century. They were detected by LIGO detectors, located in Livingston, Louisiana and Hanford, Washington in 2015 for the first time. The signals detected by the detectors originated from black hole mergers and have started the age of gravitational wave astronomy. Another exciting source of gravitational waves originate from asymmetrically rotating neutron stars. These gravitational wave signals are expected to be continuous monochromatic waves at twice the rotational frequency of the neutron star.

II. TARGETED SEARCH METHOD

The targeted search method, [2] developed by the Rome data analysis group, works by searching for continous g.w.s at twice the rotational frequency of a neutron star. This method relies on electromagnetic observations of neutron stars to obtain pulsar parameters. The signal is expected to have 5 Fourier components at the frequencies $\omega_0, \omega_0 \pm \Omega, \omega_0 \pm 2\Omega$, where ω_0 is the intrinstic frequency and Ω is the sidereal angular frequency, and the noise.

The first step is to generate a pulsar signal to inject into the data. This is done using a pulsar template that contains all the known pulsar parameters. Once the signal is injected, barycentric and spin-down corrections must be made. The first is due to the doppler effect, which occurs due to the detector velocity. This velocity is composed of the rotational velocity of the Earth as well as the Earth's orbital velocity around the sun. The spin-down correction is applied as well as smaller corrections for the Romero and Einstein delay. Then the expected form of the data is,

$$\mathbf{X} = H_0 e^{J\Phi}(H_+ A^+ + H_\mathbf{x} A^\mathbf{x}) + \mathbf{N} \tag{1}$$

where $A^{+/\mathbf{x}}$ is the 5-vector signal template. Then we can compute the amplitude estimator for both polarizations.

$$\hat{H}_{+/\mathbf{x}} = \frac{\mathbf{X} * A^{+/\mathbf{x}}}{|A^{+/\mathbf{x}}|^2} \tag{2}$$

Using the amplitude estimators and the signal templates one can finally compute the detection statistic.

$$S = |A^+|^4 |\hat{H}_+|^2 + |A^\mathbf{x}|^4 |\hat{H}_\mathbf{x}|^2 \tag{3}$$

The same process can be done on just noise in order to create a noise only distribution from which one can compute a detection statistic for just the noise. You can then determine the false alarm threshold, 90 %, in order to determine if the detection statistic corresponds to an interesting candidate.

A. Multi-Pulsar Analysis

By using an ensemble of pulsars, we investigate if by combining the detection statistic of individual pulsars we can improve the detection efficiency of these g.w.s. To begin, we randomly generated 9 pulsar templates with randomly selected parameters. The parameters were frequency, spin-down, sky position, and orientation angle. From each pulsar we generated 4 signals with different optimal SNRs (2, 6, 16, 30). Using O1 LIGO Hanford data, we injected the 36 signals into 1.5 months of data, corresponding from the 1st of December 2015 until the end of the O1 run in January 2016.

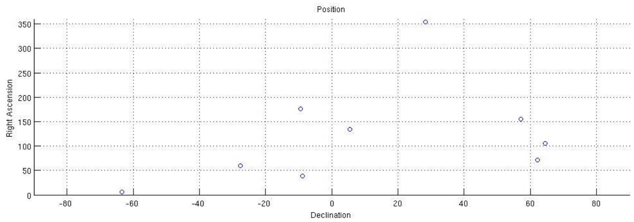

FIG. 1. Pulsar Position

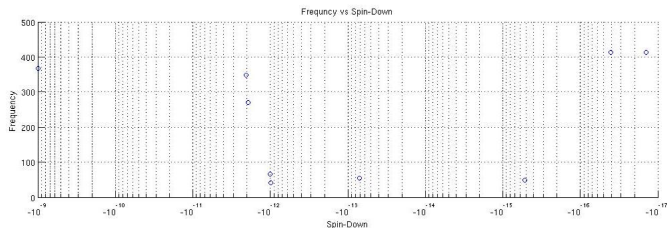

FIG. 2. Frequency vs Spin-Down

The two methods for combining the detection statistics were a simple summation and a linear combination weighed by the sensitivity of the detector at the frequency of each signal. For each of these signals a corresponding noise only distribution detection statistic was computed as well. An average is then computed. The summation method was simple, just sum all the detection statistics

$$DS = \sum_{n=1}^{9} ds_n \tag{4}$$

The summation ratio is then computed by dividing the sum by average noise detection statistic

The weighted linear combination is computed as follows,

$$DS = \sum_{n=1}^{9} \frac{ds_n}{S_h(f)} \tag{5}$$

with the linear combination ratio computed by dividing the sum by the average noise detection statistic. These

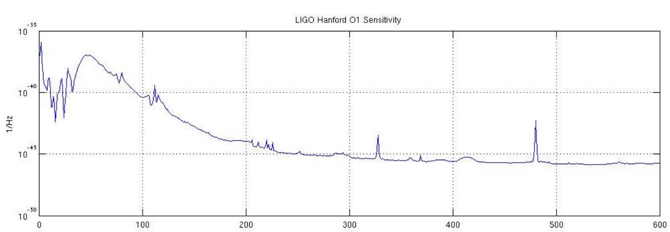

FIG. 3. LIGO Hanford O1 Sensitivity

ratios were compared to a single pulsar analysis ratio and noise only ratio in order to quantify an improvement.

III. RESULTS & CONCLUSIONS

After computing the four ratios for summation, linear combination, single pulsar, and noise only we can plot them against the optimal snr of the pulsar templates.

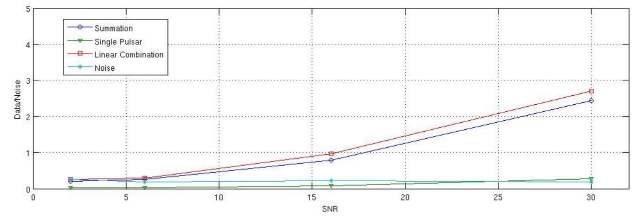

FIG. 4. Optimal SNR

The optimal snr does not take into account the orientation of the pulsar. This reduces the snr by a factor of .4 and can be done by averaging the signal amplitude over the parameter space. The snr is further reduced by taking into account the science times of the detector. These are designated times during the run when the detector was working optimally for science experiments. The LIGO Hanford detector had a duty cycle of 62 % during the O1 run. The reduces the snr by a factor of $\sqrt{.62}$.

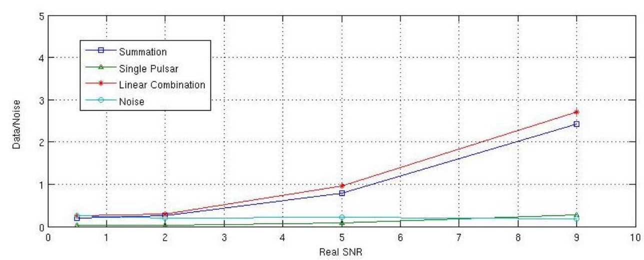

FIG. 5. Real SNR

The results show that by simply combining the detection statistics from an ensemble of known pulsars the detection efficiency increases. The best results were for the weighted linear combination, which was expected as this method gives bigger weights to signals whose frequencies the detector is most sensitive to. This method is greatly suited for signals that individually would be right below the false alarm rate. As seen from the plots an individual pulsar would not be detected except for the highest snr. This method would also be insensitive to signals with a low snr. The preliminary study has shown that using an ensemble of known pulsars one can increase the detection efficiency of continuous gravitational waves making it worthwhile to continue this method of detecting signals. To continue the study one could apply the method the real pulsar like the Crab and Vela pulsars. Another interesting approach would be to expand the method to the Rome's group narrow-band method [1] that allows the the frequency and frequency derivative to drive a little bit. Another straightforward improvement would be to increase the number of signals for a bigger and better statistical sample. A method of combining that could be used is a linear combination using the spin-down limit.

[1] P. Astone, A. Colla, S. DAntonio, S. Frasca, C. Palomba, and R. Serafinelli *Method for narrow-band search of continuous gravitational wave signals*. Physical Review D 89, 062008 (2014)

[2] P. Astone, S. DAntonio, S. Frasca, and C. Palomba *A Method for detection of known sources of continuous gravitational wave signals in non-stationary data* Class. Quantum Grav. 27 (2010) 194016

[3] Xilong Fan, Yanbei Chen, and Christopher Messenger *Detection of gravitational waves from an ensemble of known pulsars*