Thermal Noise in Non-Equilibrium Steady State

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(August 2014)

Abstract

Gravitational wave detectors are working to increase their sensitivity. Thermal noise is one of the greatest barriers to achieving necessary sensitivity. In an effort to reduce thermal noise, KAGRA intends to bring their test masses down to cryogenic temperatures. The powerful lasers used in these detectors will introduce heat into these mirrors. The heat will need to be continuously taken out in order to maintain cryogenic temperatures. This paper is a progress report on the current status of the potential thermal noise problems introduced by this heat transfer.

Introduction

For hundreds of years, astronomers have looked at the skies and wondered about the secrets of the universe. All information about the universe that humans have pieced together has been observed, directly and indirectly, through the electromagnetic spectrum. Through the different bands of radio, infrared, visible, ultraviolet light, we've been able to see different aspects of stars and other astronomical phenomena and bring them together to further our understanding. However, the EM spectrum has its limits, and it has been our *only* primary source of data collection for the universe at large.

In his 1916 paper on the Theory of Relativity, Albert Einstein predicted the existence of gravitational waves [1]. These are small ripples in spacetime, caused by massive objects, such as neutron stars and black holes. Gravitational waves would be an entirely new spectrum of waves to detect and analyze. They could tell us a great deal about events such as the gravitational collapse of stars, asymmetrical pulsars, binary inspirals, and the stochastic background. However, gravitational waves are predicted to have incredibly low frequencies. Due to a plethora of low frequency noise sources, directly detecting gravitational waves is incredibly difficult.

In 1974, Russell Hulse and Joseph Taylor discovered the PSR B1913+16 pulsar. Their observations showed that its orbit was gradually contracting due to emission of gravitational waves as predicted by Einstein [2]. Although indirect, this was the first experimental evidence of gravitational waves.

Now, collaborations such as KAGRA (Kamioka Gravitational wave detectors), LIGO (Laser Interferometer Gravitational wave Observatory), and Virgo are attempting to directly detect these ripples in spacetime. They are creating specialized Michelson interferometers in order to detect the distortion in spacetime by the pathlength difference of a powerful laser as a gravitational wave passes through. There are various sources of noise that prevent these detectors from having the necessary sensitivity, such as seismic, quantum, and thermal noise.

In an attempt to reduce thermal noise, KAGRA plans on lower the test masses' temperatures to cryogenic levels. This paper addresses the current efforts in understanding how thermal transfer affects the noise of the system.

Thermal Noise and the Fluctuation-Dissipation Theorem

Thermal Noise is due to heat turning into mechanical energy in a system. It is very difficult to measure directly, so the Fluctuation-Dissipation Theorem is used instead. In accordance to the Fluctuation-Dissipation Theorem, if a system is in thermodynamic equilibrium, the amount of thermal energy that dissipates from the system into mechanical energy is the same amount of mechanical energy that can enter a system as thermal energy [3]. This is a useful principle that we use to measure thermal noise indirectly. We determine thermal noise by the amount of energy leaving the system.

Say we have a material whose resonance frequency mode we can measure. We can determine the thermal noise in the material by first finding the Q (i.e. Quality Factor). By exciting the material at its resonance mode and then measuring the rate at which the excitation dies away, we can calculate the damping ratio ζ, the attenuation rate α, or the exponential time constant τ to find the Q

$$Q = \frac{1}{2\zeta} = \frac{\omega_0}{2\alpha} = \frac{\tau\omega_0}{2},$$

The reciprocal of the Quality Factor is the Dissipation Factor, which is a measure of the rate of energy lost in a mode of oscillation, in our case, mechanical energy.

The KAGRA collaboration intends to lower its test masses to cryogenic levels in order to lower thermal noise. However, there are large, high-powered lasers incident on these test masses, introducing a source of heat, and requiring for a near constant flow of heat out of the test masses while the detector is on-line. This means that the masses will not be in thermodynamic equilibrium, which is required for the Fluctuation-Dissipation Theorem. However, for these masses, a non-equilibrium steady state is assumed. Assume the rate of thermal transfer is constant. If you consider a number of infinitely small slices along the mass and connecting wire, each slice will not fluctuate in temperature. A local thermodynamic equilibrium can be assumed. With these assumptions, the Fluctuation-Dissipation Theorem is still valid.

This system was theoretically and numerically modeled and found that there is almost no thermal noise added to the system from the transfer of heat [4].

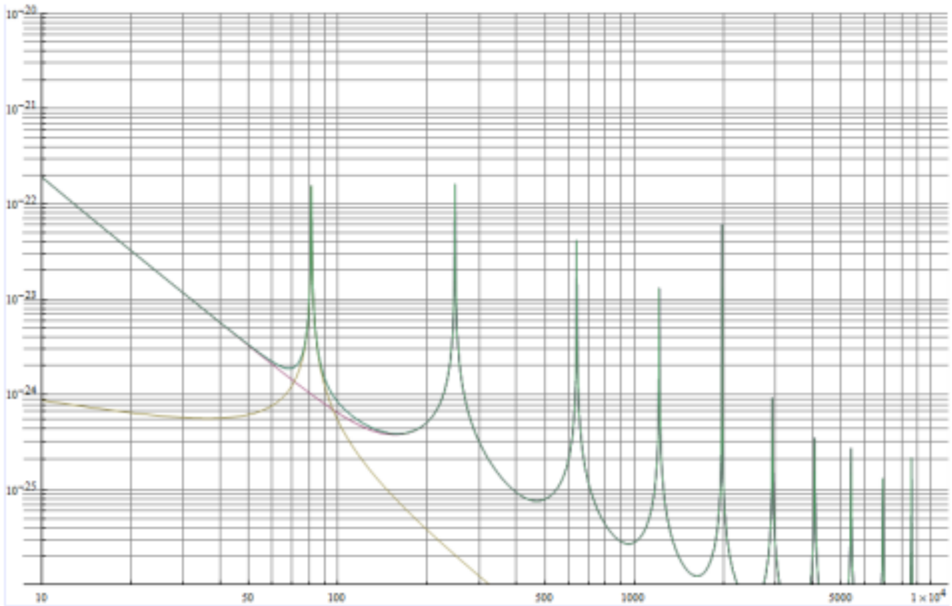

Fig. 1 The green line is the suspension noise with the temperature gradient. The blue line is without the temperature gradient.

The ratio of RMS amplitudes with and without the temperature gradient was calculated. The temperature difference was about 4K, so the ratio is about 10 percent.

$$R = \frac{\langle x^2 \rangle^{NESS}}{\langle x^2 \rangle^{EQ}} - 1$$

$$R = C_1 \Delta T + C_2 (\Delta T)^2$$

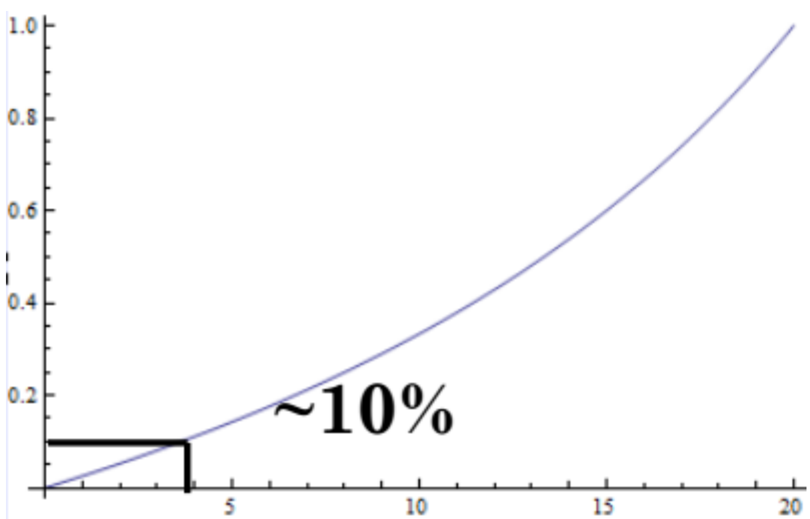

Fig. 2 Calculated RMS

This is the theoretical justification that having heat flow in a non-equilibrium steady state will not seriously impact the overall noise level. Now just needs to be verified experimentally [4].

Method/Current Progress

We used a wire of tungsten. We used a michelson interferometer to measure the Q values of the wire.

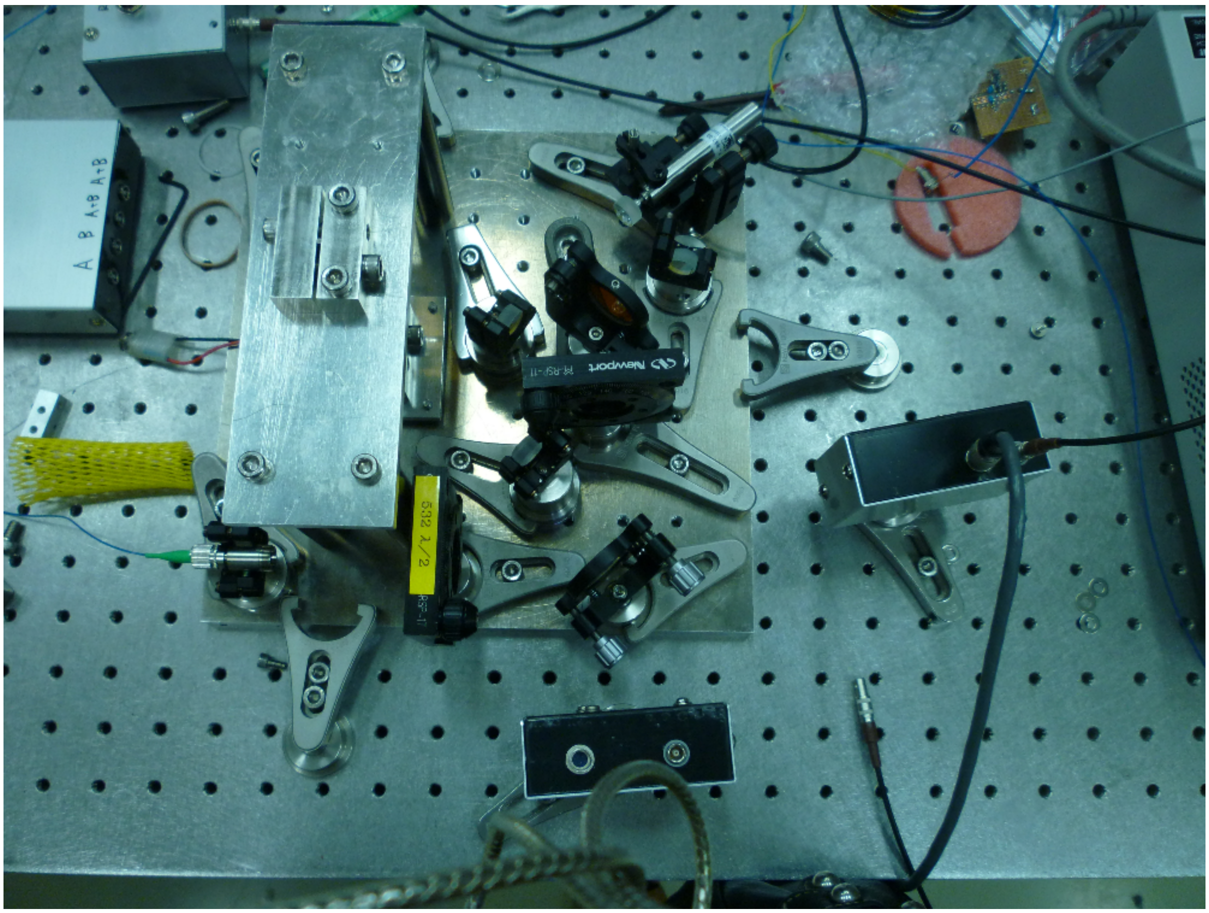

Figure 3

We attached two aluminum bars to the tungsten wire, a small mirror and magnet on each side of each bar.

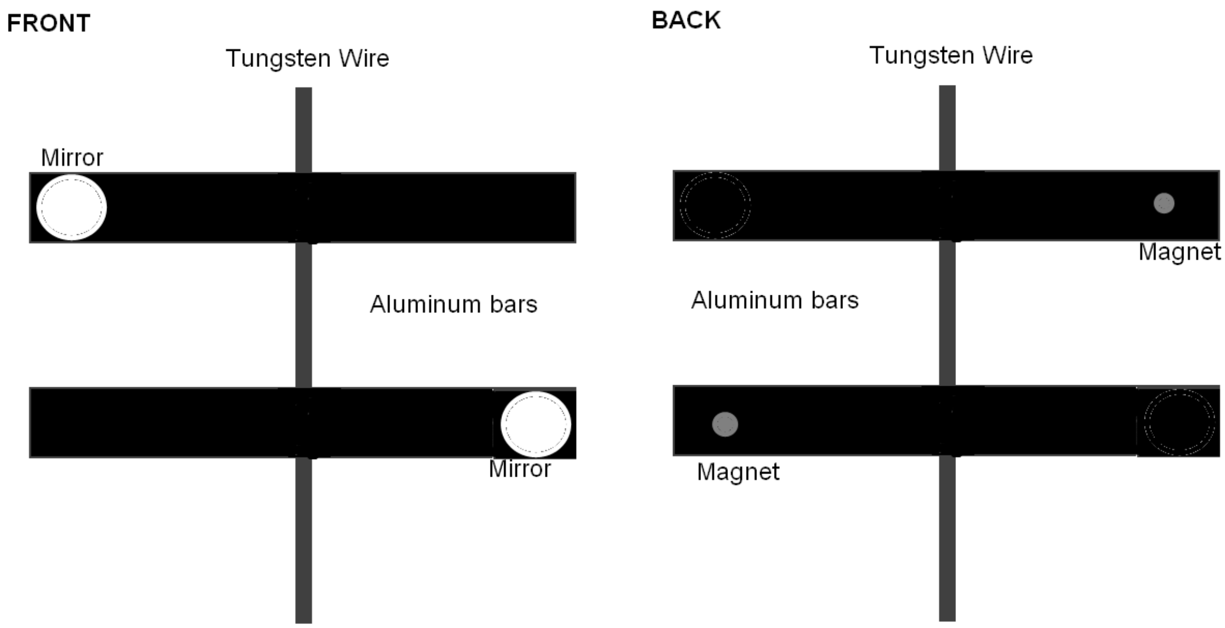

Figure 4

As the wire twists, the length of the laser beams reflecting off the mirrors will oscillate as the aluminum bars oscillate with the tungsten. In order to measure the amplitude of these oscillations, the pico-mirror is programmed to adjust so that the laser path stays constant. By observing the movement of the pico-mirror which is connected to an oscilloscope, we can determine the amplitude.

There are two separate resonant modes that we observed, the Same Phase mode, and the Inverted Phase mode, as shown in Figure 5.

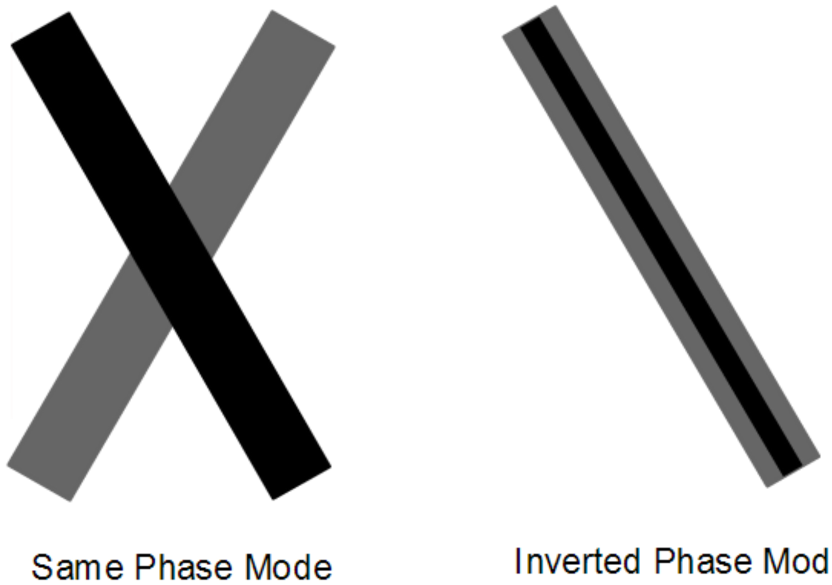

Figure 5

In order to find the frequency of these modes, we need to excite the bar to twist at certain frequencies and observe the amplitude of the oscillations. To do this, magnetic coils connected to a waveform generator were placed in front of the magnets attached to the aluminum bars. Using the waveform generator, we can excite the tungsten at different frequencies with the coils exciting the bars in phase with each other (Same Phase mode) and out of phase with each other (Inverted Phase mode). By sweeping through multiple frequencies and exciting the bars in and out of phase, we found the resonant modes.

Same Phase Mode	69.11 Hz
Inverted Phase Mode	134.39 Hz

To first measure the Q values of the tungsten without heat, the interferometer was placed in vacuum to obtain the highest possible Qs. Then we excited the tungsten at its resonant modes, turned off the coils and measured how long it took for the oscillations to dissipate. The waveform of the oscillations has a function of:

$$f(t) = Ae^{\frac{-\pi\omega_0 t}{Q}}$$

Where A is a constant of proportionality, ω_o is the resonance frequency, and Q is the quality factor.

From the oscilloscope, we can obtain the sine and cosine components of the function, which are in terms of voltage. To find the absolute value of the voltage:

$$V = \sqrt{V_{sin}^2 + V_{cos}^2}$$

Then we plotted voltage versus time for both modes and found the line of best fit.

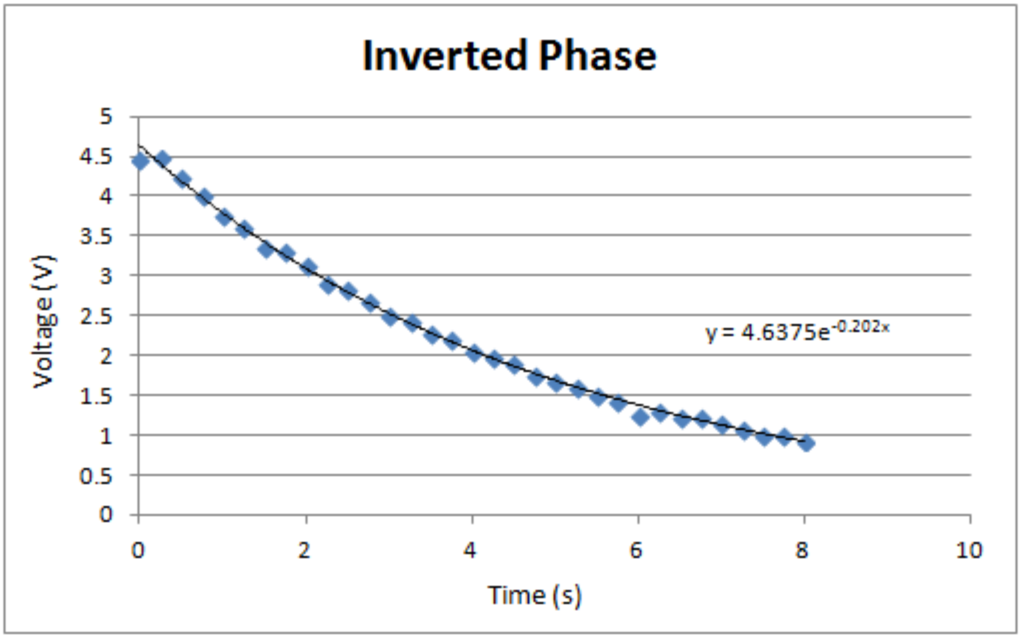

Fig. 6 Oscillation decay of Inverted Phase Mode

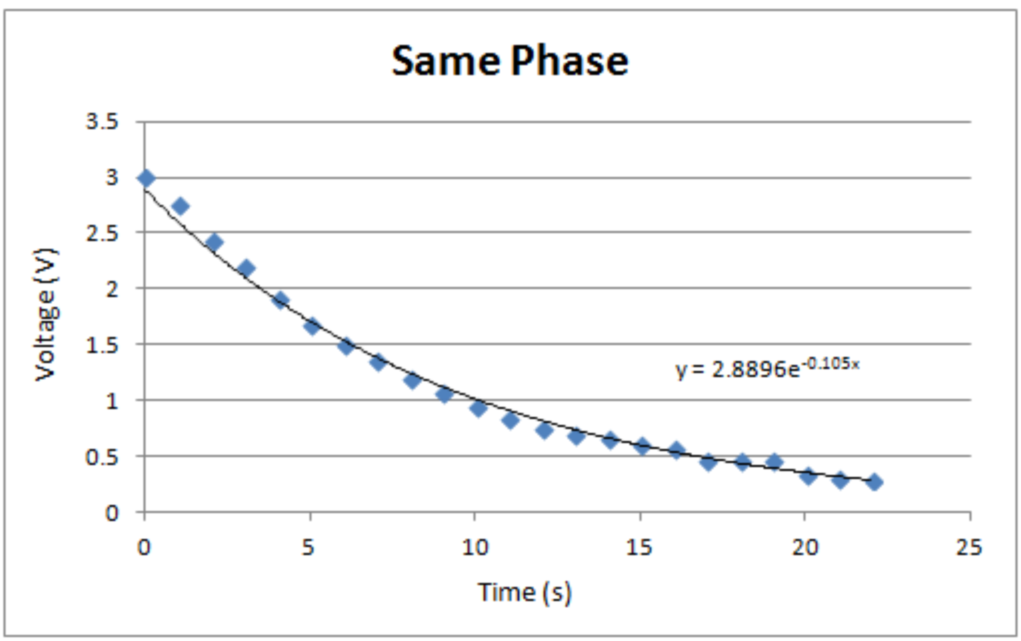

Fig. 7 Oscillation decay of Same Phase Mode

From these lines of best fit and the waveform function, we can calculate each mode's Q value. Recall:

$$f(t) = Ae^{\frac{-\pi\omega_0 t}{Q}}$$

We are only trying to find the Q values, so we are only interested in the constants in the exponent. So for Inverted Phase:

$$\frac{-\pi\omega_0}{Q} = -0.202$$

$$\frac{-(3.14159)(134.39Hz)}{Q} = -0.202$$

$$\frac{(3.14159)(134.39Hz)}{0.202} = Q$$

$$2090 = Q$$

We similarly found the Q value for the Same Phase Mode. Our results:

Same Phase Mode	2068±3
Inverted Phase Mode	2090±2

Uncertainty calculated in gnuplot.

These will serve as our basis for comparison with the Q values of the tungsten when it is in nonequilibrium steady state.

Future Plans

The heater and temperature control systems are almost completed. Within a few months, the heater will be attached and monitored to achieve nonequilibrium steady state. Then a new Q value will be found for each resonant mode and compared with the original Q values through numerical analysis.

Conclusion

In an effort to increase sensitivity for gravitational wave detection, the thermal noise problem must be addressed. The KAGRA collaboration intends to bring their mirrors down to cryogenic temperatures. However, because to the high powered lasers that will be used, heat will need to be transferred out of the mirrors in order to maintain cryogenic temperatures. This transfer needs to be investigated in order to ensure that proper levels of sensitive will still be procured. Due to the relationship between thermal noise and mechanical loss via the Fluctuation-Dissipation Theorem, the idea of nonequilibrium steady state was developed in order to look at the potential effects of heat transfer. Theoretically, nonequilibrium steady states seem to be a viable solution to the heat transfer problem in cryogenic mirrors. We are currently in the process of verifying this experimentally.

References

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