The Dance of Death of Binary Black Hole Systems

Ben Hubbert, Carleton College '14

Abstract: Binary Black Hole systems are a major potential source for gravity waves visible to Advanced LIGO and other GW detectors. Data analysts have developed several parameter-estimation codes to determine physical characteristics of binary black-hole systems from the gravitational radiation they emit. The LSC tests these algorithms using simulated gravitational-wave data generated by analytical post-Newtonian approximation. The NINJA (Numerical INJection Analysis) project enlisted numerical relativists to provide LSC data analysts with highly accurate gravitational waveforms generated by numerical simulations; however, the NINJA project imposed no minimum requirements on the accuracy of these waveforms. We check the accuracy of some waveforms from the NINJA-2 waveform library by running a Markov Chain Monte Carlo parameter-estimation code on multiple numerical waveforms with identical parameters and comparing the results. We show that all of the different waveforms can be recovered with similar values for most astrophysically important parameters such as distance, sky position, and component mass, but that different waveforms are also recovered with different polarizations. We recommend that further study be conducted across a wider range of different waveforms before a decision is made on whether to implement concrete accuracy requirements on NINJA waveform submissions.

1. Introduction

According to the theory of General Relativity, any accelerating mass will produce gravitational waves – distortions in space-time that propagate through space at the speed of light. Furthermore, the gravitational-wave signals from astronomically massive objects such as neutron stars and black holes can theoretically be detected by interferometers such as the Laser Interferometer Gravitational Wave Observatory (LIGO). Binary Black Hole (BBH) systems – pairs of co-orbiting black holes – are one possible astrophysical source for gravitational waves strong enough to be visible in modern or near-future detectors. General Relativity predicts that as two black holes orbit one another, they will radiate away some of their energy in the form of gravitational waves, and thus gradually spiral inward at increasing speeds before eventually merging[1]. The gravitational waves produced by BBH systems are not only uniquely powerful but also have a distinctive evolution over time – as the system evolves towards the final merger, the frequency and amplitude of the emitted gravitational radiation will increase and finally peak just as the two black holes merge. These distinctive features make BBH systems ideal candidates for detection by LIGO, Virgo, and other GW detectors.

In addition to confirming the long-theorized existence of gravitational waves, GW detectors have the potential to provide a wealth of information on gravitational-wave sources, and thereby complement conventional and radio telescopes as an

---

[1] Benjamin Aylott et al.: Testing gravitational-wave searches with numerical relativity waveforms: Results from the first Numerical INJection Analysis (NINJA) project. arXiv:0901.4399v2 [gr-qc]

important tool for astronomy and astrophysics. To determine information about a GW signal's source, the LIGO Scientific Collaboration (LSC) relies on parameter-estimation algorithms such as lalinference, which can determine the sky position, distance, and component masses and spins of GW sources. To find this information, lalinference employs a method known as a matched template search: it generates a simulated waveform referred to as a template, then takes the inner product of the template and the GW signal (or, more accurately, the LIGO data at the chosen trigger time, which contains both a GW signal and noise) in the frequency domain. The template for which the inner product is highest bears the closest resemblance to the actual waveform, and therefore the parameters used to generate it are the most likely parameters of the real waveform's source.

The primary difficulty inherent in the matched template search technique is the size of the parameter space, which spans up to 15 dimensions. For each point in the parameter space we search, we must compute a template and take an inner product, both of which are non-trivial tasks; if we were to explore the entire parameter space with a grid search, the number of points to map would grow exponentially with each added parameter. Therefore, lalinference uses several methods to narrow down its search to the most likely volumes in the parameter space. The method used for our investigation is known as Markov-Chain Monte Carlo (MCMC).

After choosing a random starting point $\vec{\theta}$ in the parameter space, the MCMC algorithm randomly selects a new point $\vec{\theta}'$ within a certain distance of its current location. The MCMC algorithm then uses templates to compute the likelihoods $p(\vec{\theta})$

and $p(\vec{\theta}')$ that each of these points represents the waveform's parameters, and the ratio between these two likelihoods determines the probability of the algorithm jumping to the new point. After it randomly determines whether to jump to the new point or remain on the old one, the algorithm randomly selects another point and repeats the process. The jump probability $P(\vec{\theta} \rightarrow \vec{\theta}')$ satisfies a condition known as the detailed balance:

$$p(\vec{\theta})P(\vec{\theta} \rightarrow \vec{\theta}') = p(\vec{\theta}')P(\vec{\theta}' \rightarrow \vec{\theta})$$

where $P(\vec{\theta}' \rightarrow \vec{\theta})$ is the probability of jumping in the reverse direction. This condition ensures that after many iterations – several million in some cases -- the MCMC algorithm will converge on the point with the highest likelihood.

As this process repeats, the points sampled by the algorithm form a map of the parameter space, where the density of the sampled points at any given location is proportional to the probability density there. Essentially, the MCMC algorithm allows us to construct a probability density function for the parameters of a GW event.

Until the first science run by Advanced LIGO and Advanced Virgo in 2014, the LSC will not possess a GW detector with sensitivity sufficient to resolve GW signals from any known astrophysical source. In order to determine the sensitivity and effectiveness of search and parameter-estimation algorithms in the absence of real data, LSC investigators rely on injections – simulated GW signals added to computer-generated noise or real data from GW detectors. In the past, most of these injections have used waveforms generated by Post-Newtonian (PN) approximation. PN approximation uses a Taylor-series expansion of Einstein's general-relativity equation about the normalized velocity term to find approximate analytical solutions to Einstein's equation. The key advantage of PN approximation is that it can be done analytically and relatively quickly; however, PN approximations are by nature inexact, and the waveforms they produce are therefore inaccurate. In the specific case of BBH systems, the approximation becomes increasingly inaccurate as the two black holes approach one another because their speed increases, and speed (normalized by the speed of light) is the PN approximation's expansion parameter. Therefore, PN approximations are most inaccurate at the point where the two black holes merge – which is also the point at which the GW signal is loudest and most easily detected.

To address these problems, the Numerical INJection Analysis (NINJA) project was launched in 2008. NINJA is a collaboration between numerical relativists and LIGO data analysts. The NINJA project's goal was to generate injectable GW waveforms based on numerical solutions to Einstein's equation, then study the properties of these waveforms in a GW detector. In particular, it was conceived to determine whether numerically-generated waveforms could be detected and analyzed using templates generated with PN approximation, since these templates take much less time to compute than numerically-generated waveforms. Several different numerical relativity groups joined NINJA and submitted waveforms using their own simulation codes; NINJA then made these waveforms available to data analysts for use in injections. Currently, the NINJA project is in the process of compiling its second set of waveforms, known as NINJA-2. For the initial phases of the NINJA project, only simulations of BBH systems were accepted and no accuracy requirements were set on the simulations.

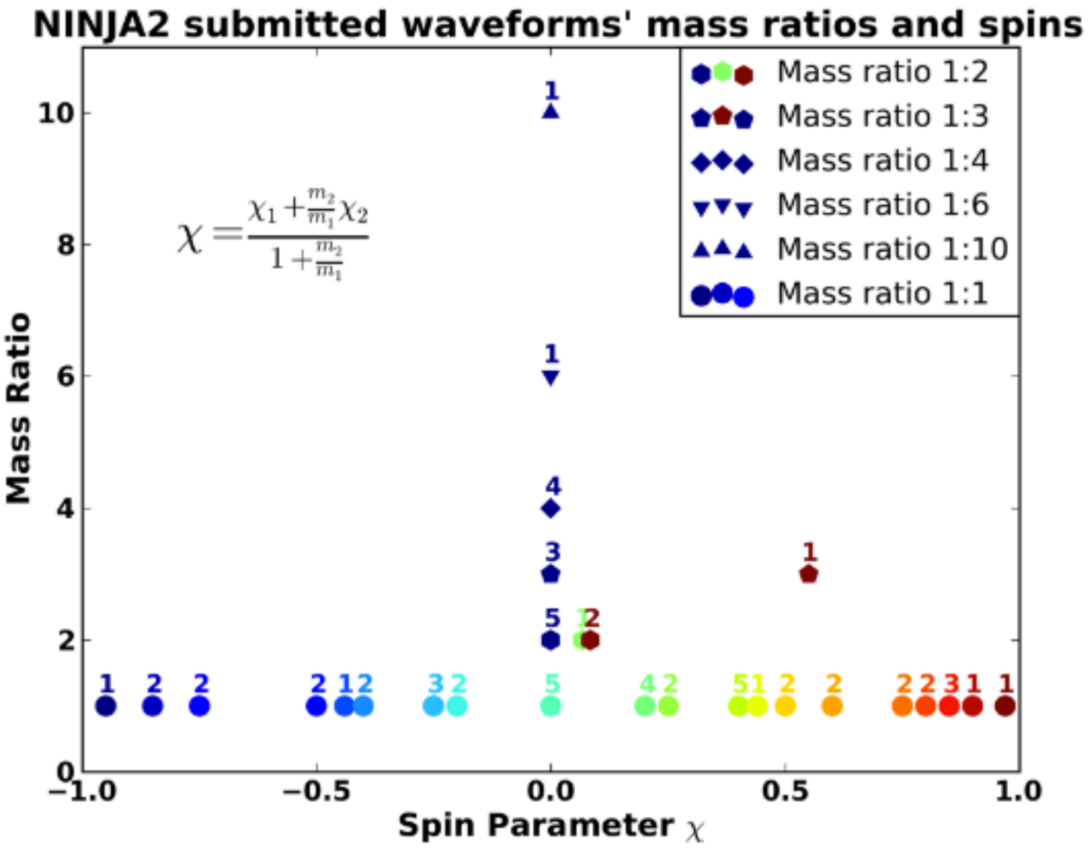


Figure 1. *NINJA-2 waveforms by mass ratio (ratio between the masses of the black holes in the BBH system) and spin parameter.*

The waveforms produced by NINJA have the potential to be far more accurate than waveforms created with PN approximation, but their actual accuracy depends on the code used to run the simulation. Factors such as grid resolution, order of integration, and the methods by which singularities, event horizons, and the boundaries of the simulated space are handled within the program affect the accuracy of each simulation. In addition, the waveforms used in the NINJA project are hybrid waveforms – combinations of a PN approximation of the early inspiral phase of a BBH system and a numerical simulation of the late inspiral, merger, and ringdown phases. Hybrid waveforms require far less time to compute than pure numerical-relativity waveforms, but some researchers have suggested that the hybridization process may be an even greater source of error than the numerical simulation[2]. Because the NINJA project set no accuracy requirements on submitted waveforms, there is currently no guarantee that all of the waveforms in the NINJA catalog are sufficiently accurate simulations of a BBH inspiral.

We attempt to check the accuracy of the NINJA-2 waveforms by making numerical-relativity injections with known parameters, and then recovering these parameters with the lalinference parameter-estimation code. Earlier attempts to estimate the parameters of numerically generated injections show that analytically-generated templates are not yet fully accurate when applied to numerical-relativity waveforms, but we can set a lower bound on the required accuracy by comparing different injections made with identical parameters. These comparisons can in turn be used to make NINJA's numerical simulations more accurate, which will allow us to improve our ability to detect them and estimate their parameters.

2. Experimental Methods

---

[2] Ilana MacDonald, Samaya Nissanke, and Harald P. Pfeiffer: Suitability of Post-Newtonian/Numerical-Relativity Hybrid Waveforms for Gravitational Wave Detectors. Class.Quant.Grav.28:134002,2011

We ran on two different types of injections in this experiment. First, we examined the blind injection test data set released by NINJA, in order to ensure that we had installed and configured lalinference properly. This was a series of frames released by NINJA, containing several numerical-relativity injections at unknown times against a background of Gaussian noise. By the time we began our experiment, other groups had already used search algorithms to find several injections. The blind injection set was an ideal test because other groups had already run lalinference_mcmc on the data and produced results with which we could compare our data. A secondary purpose of this part of the experiment was to add to the collection of data available on the blind injection set, in order to help other groups check their results.

The data from each lalinference_mcmc run was parsed by cbcBayesPostProc, a python script included in LAL's pylal library. cbcBayesPostProc automatically created plots of the data and posted them to a webpage for public viewing.

Although the blind injection frames could be parsed by lalinference as they were downloaded, the waveforms in NINJA's injection catalogue needed to be injected into frames before we could analyze them. NINJA-2 waveforms allow the user to set the total mass, sky location, distance, and inclination of the BBH system as well as the polarization of its GW emissions. For our tests, we chose a total mass of 100 solar masses, a sky location of 20 degrees latitude and 60 degrees longitude, a distance of 1000 megaparsecs, and fixed both inclination and polarization at 45 degrees. We injected all our waveforms at the same GPS time in order to ensure that the detectors would be in the same orientation relative to the incoming GW signal for each injection. For each waveform we examined, we first used the lalapps_inspinj tool to create an xml file containing an injection from each waveform, then injected that data into empty frame files (the common file format used by the LSC to store output from GW detectors) using the lalapps_mdc_ninja tool. We used the lalinference_mcmc codes to recover the injection from the frame files in the LIGO Hanford 4km, LIGO Livingston, and Virgo detectors. The injections were made with zero noise realization – the injection frames contained a GW signal with zero noise. To conduct the estimates of noise power-spectral density (PSD) required by lalinference, we used gstlal_color_frames to create a single set of frames containing only Gaussian noise, at a GPS time well after the injections. To simplify the experiment, we used the estimated PSD of early Advanced LIGO for all three interferometers in the test, even though this PSD would not be accurate for Virgo.

For our first test, we selected the simplest waveforms: those in which both black holes in the simulated BBH system had equal mass and zero spin. There were five waveforms in total: one waveform created with Caltech and Cornell's SpEC code, one created with Georgia Tech's MayaKranc code, one created with the Llama code, and two created with the BAM code. These last two were created with the same code, but hybridized to PN waveforms computed with Taylor expansions of differing order – one first-order expansion and one fourth-order expansion. We were particularly interested in this pair of waveforms as they would provide a clear indication as to how much error different hybridization methods could produce.

3. Results

Examining the data from our first test, we found that in most cases, the PDFs produced by the five different injections peaked in consistent locations, but their peaks were often shaped very differently and somewhat ‘messy’; see figures 2 and 3 for examples. Although the mass, distance, and sky position parameters were broadly similar across all five waveforms tested, the polarization parameters iota and psi varied widely between each waveform, including between the two differently-hybridized BAM waveforms. A quick check showed that the polarization parameters were consistent between two injections made with the same numerical waveform and nearly identical parameters. This is a significant result, since the injections were created with a fixed polarization.

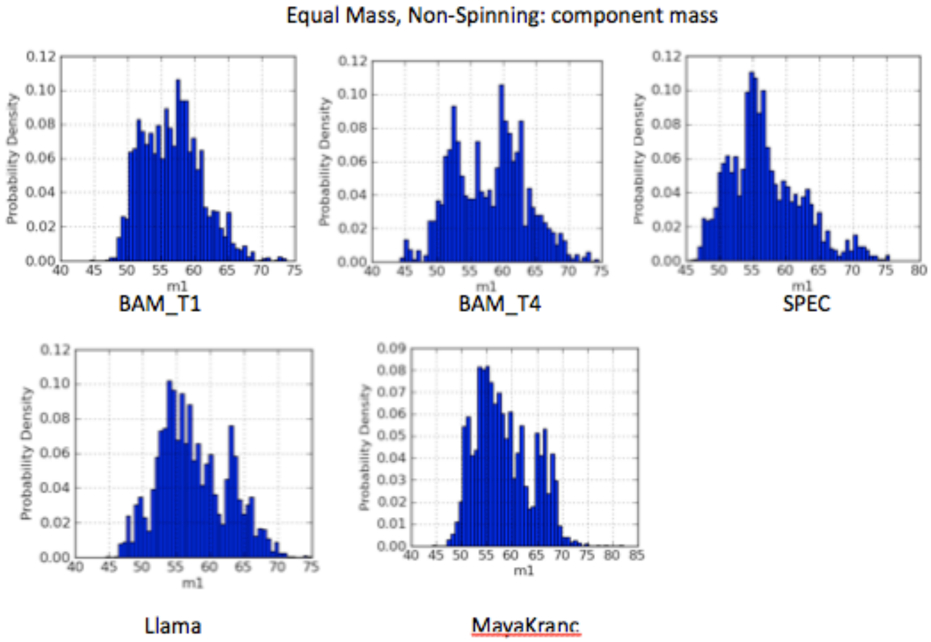


Figure 2. *Component mass PDFs for the five waveforms representing an equal-mass, non-spinning BBH system.*

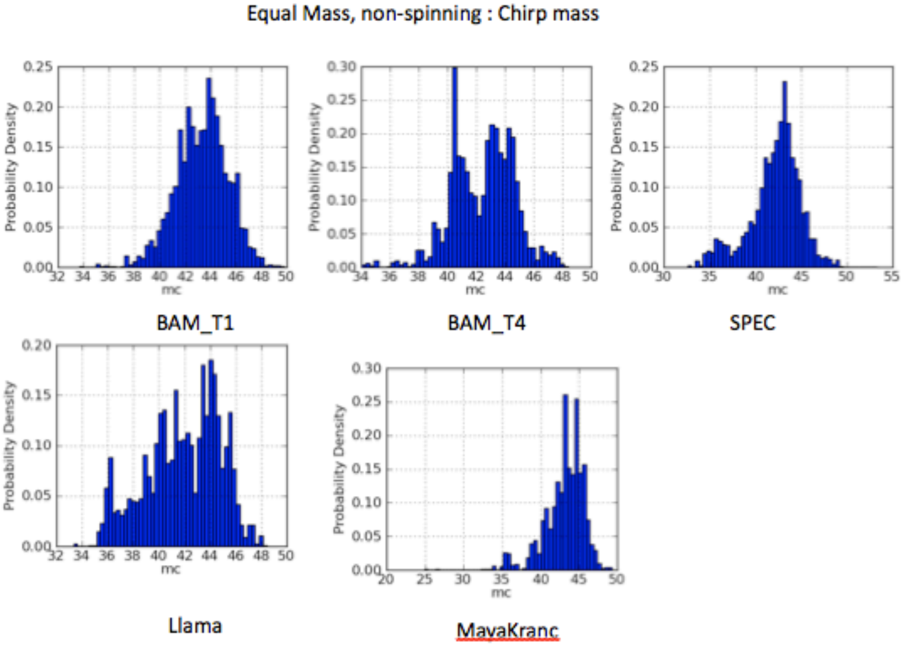
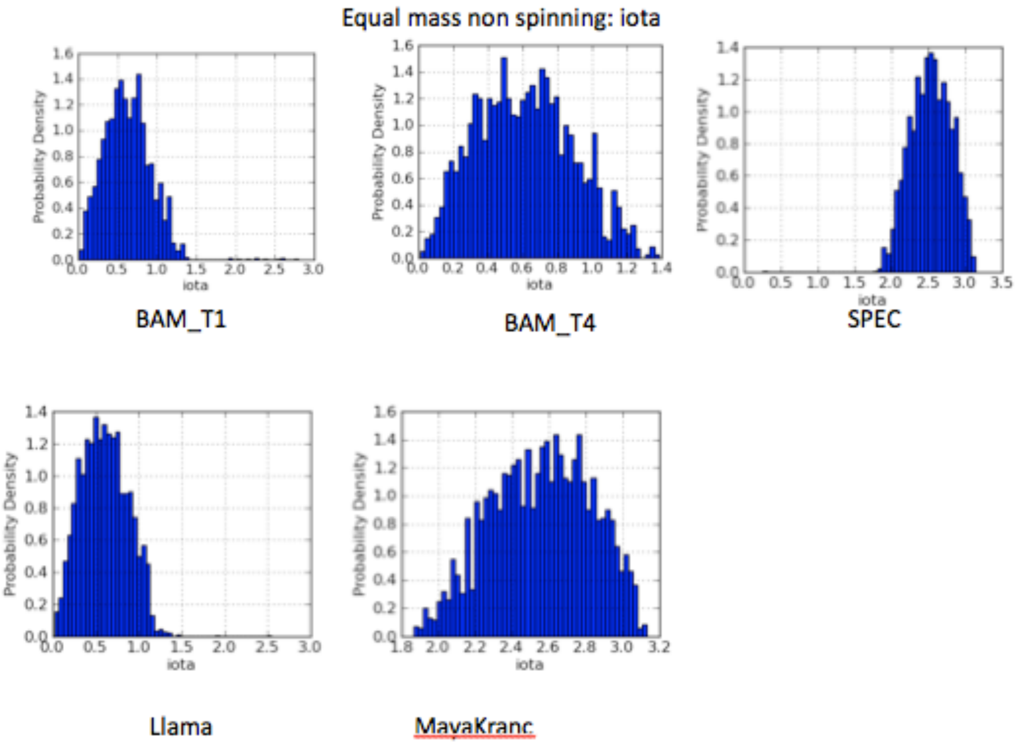


Figure 3. *Chirp mass PDFs for the five waveforms representing an equal-mass, non-spinning BBH system.*



BAM_T1

BAM_T4

SPEC

Llama

MayaKranc

Figure 4. *Iota PDFs for all five waveforms representing an equal-mass, non-spinning BBH system. Note that even though all five injections were made with the same value for Iota, the PDFs vary widely.*

Our second test explored waveforms with zero component spin and a mass ratio of 2:1. Again, there were five waveforms, two made with the BAM code and one each made with the Llama, MayaKranc, and SpEC codes. For this test, we obtained broadly similar results; although most parameters were consistent across the entire group of waveforms, the polarization parameters varied widely. Notably, the PDFs were much 'cleaner' than those of the equal mass BBH systems; although no parameters were altered between the two runs, lalinference_mcmc gathered significantly more data points during the second run than the first.

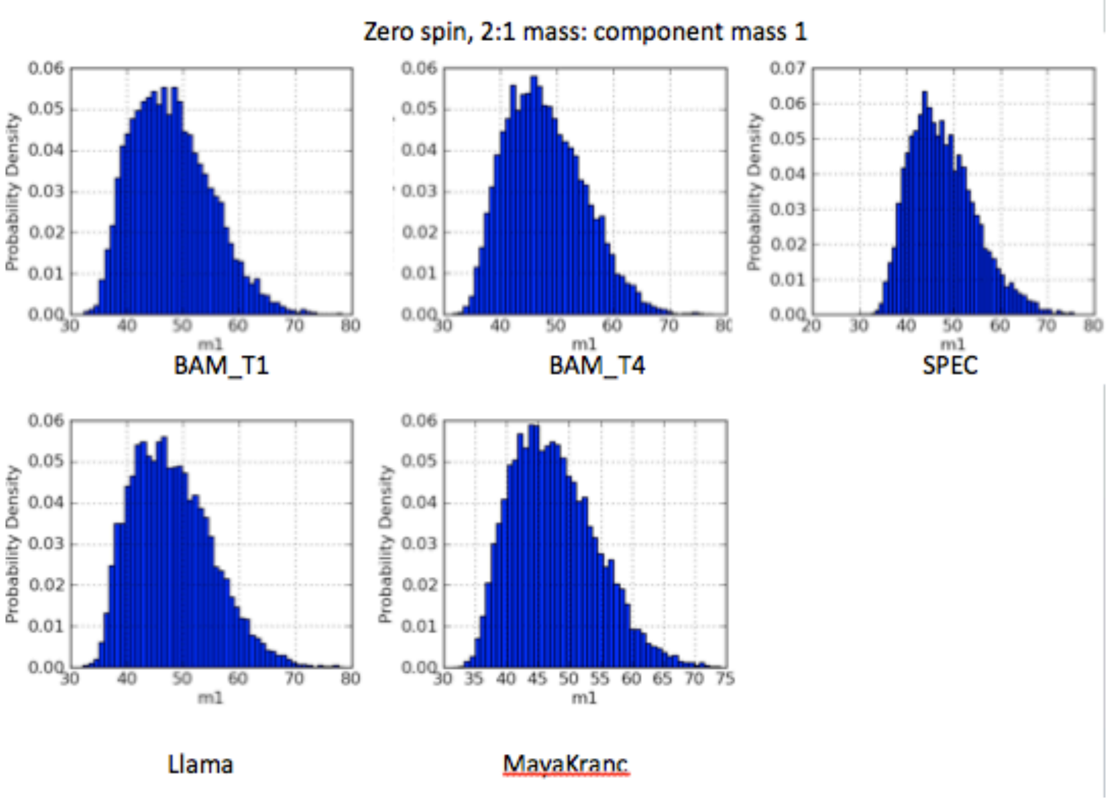


Figure 5. *Component mass PDFs for all five waveforms representing a 2:1 mass, non-spinning BBH system.*

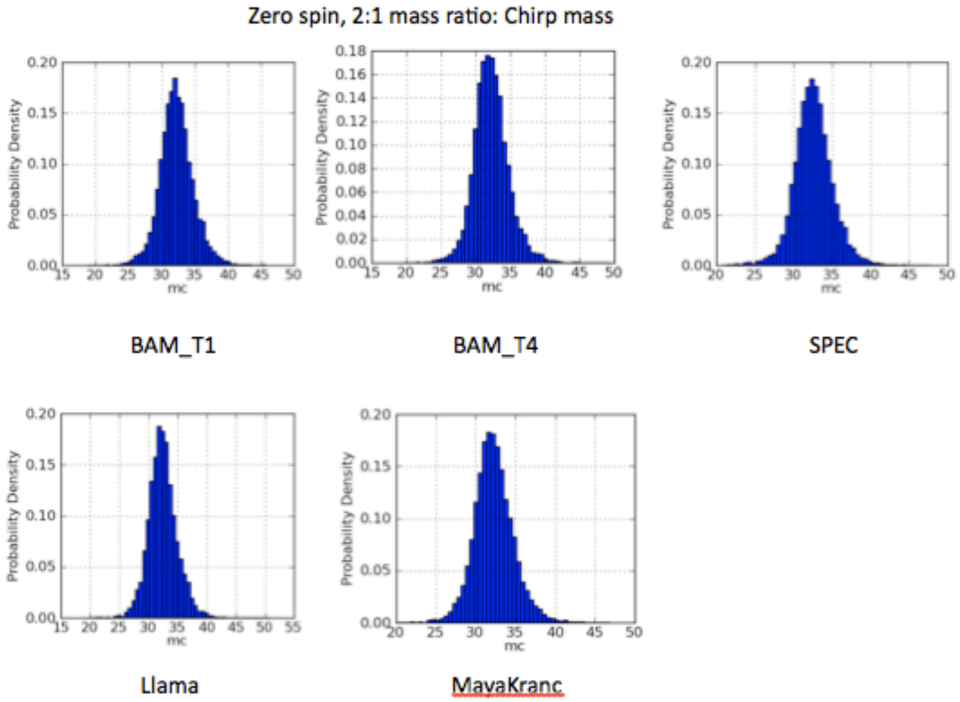


Figure 6. *Chirp mass PDFs for all five waveforms representing a 2:1 mass, non-spinning BBH system.*

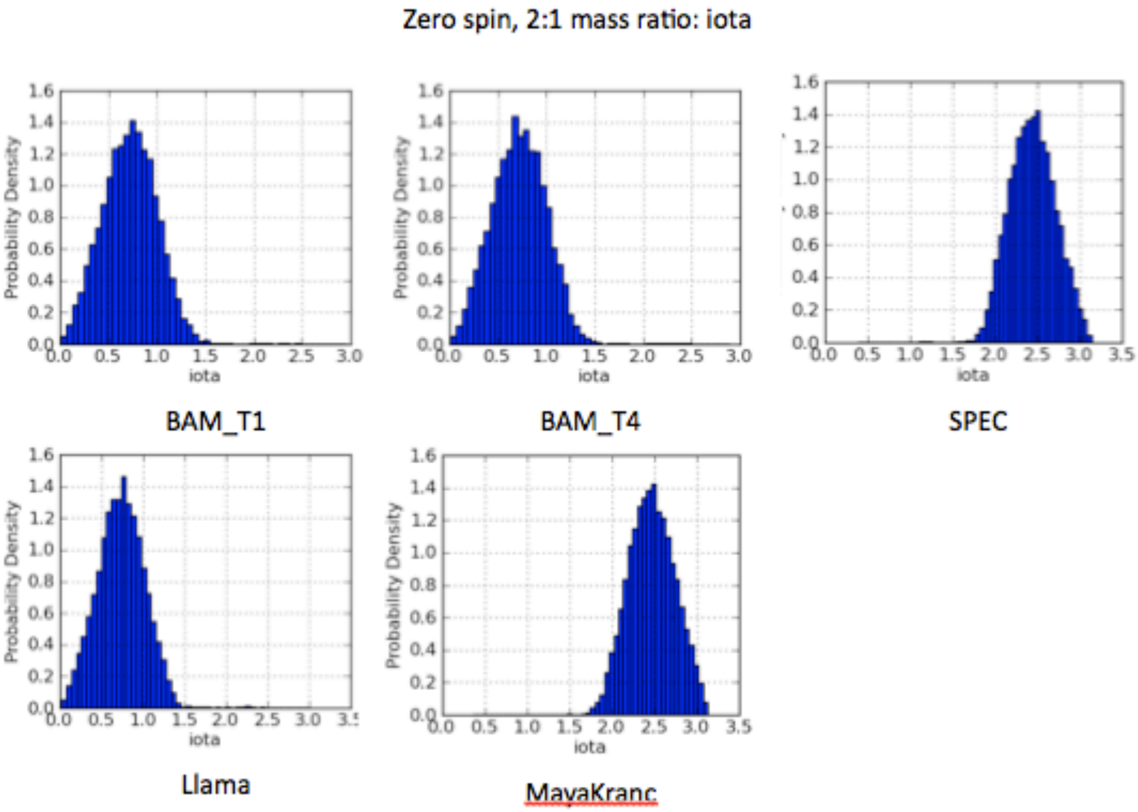


Figure 7. *Iota PDFs for all five waveforms representing a 2:1 mass, non-spinning BBH system.*

As with the equal mass ratio waveforms, the recovered parameters of these waveforms were largely consistent with the exception of the two polarization parameters.

4 . Conclusions

In many respects, the waveforms produced by the NINJA-2 project are sufficiently accurate to be used as the basis for tests of parameter-estimation algorithms. They are accurate enough that the retrieved values for component mass, chirp mass, distance and sky position are consistent across many different values. However, the polarization parameters iota and psi are not consistent between different waveforms.

This study has examined only a small portion of the NINJA library: the waveforms with relatively low mass ratios and zero component spin. In order to verify that the NINJA simulation codes are sufficiently accurate, however, we must expand our search to encompass simulations of BBH systems with high mass ratios and zero component spins. In addition, we can test each individual waveform with a wide variety of inputs for parameters such as distance, sky position, and total mass of the BBH system to ensure that each waveform remains accurate across the entire parameter space. Looking farther into the future, we can gain a better understanding of the impact of hybridization on waveform accuracy by comparing hybridized waveforms with waveforms generated using only numerical methods – which are presently impossible to construct due to computational limits.

Acknowledgements

Thanks to Bernard Whiting, Guido Mueller, Antonis Mytidis, Kristin Nichola, and the University of Florida for their generosity in offering me the opportunity to do research abroad.

Thanks to Alberto Vecchio, Ilya Mandel, and Trevor Sidery for offering me the chance to work on this project and helping me through the myriad difficulties I faced.

Thanks to Ben Aylott and David Stops for a great deal of help with computer issues.