Thermal Noise Effects in Gravitational Wave Detection: Measuring Horizontal and Vertical Mode Quality Factors of Fused Silica Fibers

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International REU 2012
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August 14, 2012

Abstract

In the Virgo interferometers, it has been determined that a monolithic fused silica fiber suspension system works fairly well to reduce thermal noise effects. However, in order to fully understand the system and where losses are occuring, full characterization of the suspension needs to be done. The purpose of the main experiments discussed is to better understand the losses associated with the clamping system used in Virgo as well as the thermal noise effects that occur from coupling between the horizontal and vertical mode oscillations (previously, only horizontal modes were considered).

Contents

Chapter 1

Introduction

1.1 Gravitational Waves

Matter tells space how to curve, and space tells matter how to move. [2]

This elegant statement by physicist John Wheeler concisely encapsulates the key concept of Albert Einstein's Theory of General Relativity. As can be seen from the statement, Einstein's theory suggested a novel way of interpreting the forces that act between bodies with mass. From the postulates in his theory, Eistein predicted the existence of gravitational waves, small amplitude oscillations in the fabric of space-time caused by the acceleration of massive bodies [1]. However, as these oscillations travel through space, their stength rapidly diminishes. As a result, the detection of these waves proves to be a great challenge, and scientists must utilize sophisticated apparatus in order to detect characterisic amplitudes on the order of 10^{-20} to 10^{-24} where the characteristic length L is defined by the distance between two masses [3]. Consequently, any noise caused by electronics, the clamping system, the bonding material, the earth's movement, etc will overpower and therefore cover up any evidence for gravitational waves. For the experiment that is the main topic of this report, the quality factor of the fused silica fibers used in the suspension system for the mirrors in the Virgo interferometer are measured to determine sources of unknown losses and gather evidence for coupling between horizontal and vertical mode resonances. This is done to better characterize the suspension system used in Virgo so that thermal noise effects may be better understood and hopefully reduced.

1.2 Use of Fused Silica Fibers in Gravitational Wave Detection

In order to reduce the effects felt by seismic noise, isolation systems are put into place in interferometers.

In the Virgo interferometer, seven filter stages compose the isolation system–"one inverted pendulum, five similar pendula, and the *marionetta* stage which holds four suspension wires," [5]. Once implemented, seismic noise becomes negligible above 5 Hz, and thus it is thermal noise that needs to be taken care of in the 5-500 Hz band [5]. To reduce thermal noise, materials with high quality factors are sought out since the thermal energy for these objects is narrowly distributed about the resonant frequency; therefore, thermal noise outside of this narrow bandwidth–the region of interest–becomes greatly reduced. In the projects discussed, further characterization of the components of this suspension with respect to the silica fibers is explored: specifically, the clamping system of the fibers and the coupling between horizontal and vertical mode oscillations.

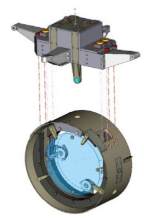


Figure 1.1: Monolithic Suspension in VIRGO [4]

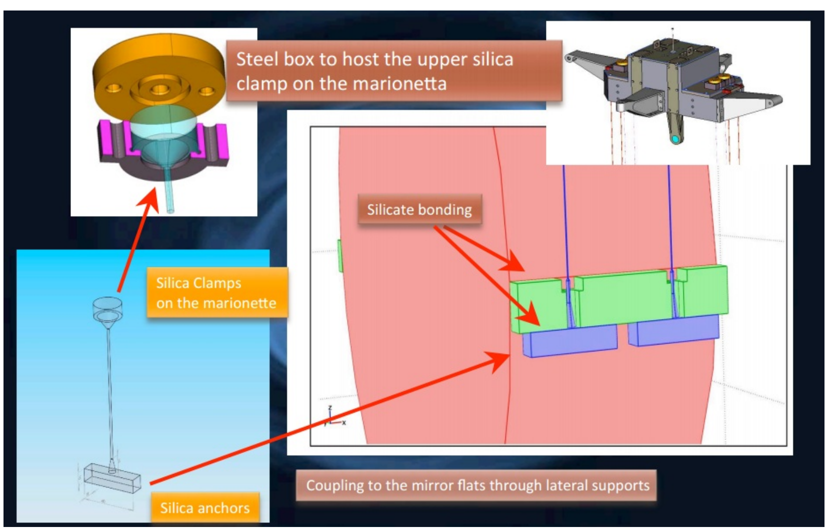


Figure 1.2: Monolithic Suspension in VIRGO [4]

Chapter 2

Loss Angle of Vertical and Horizontal Modes

2.1 Theory

As is known, the constituents of an experimental apparatus are made up of atoms–an object with an electron cloud encompassing a proton and neutron core. As a result, unless experiments are conducted at 0 Kelvin, there will always exist flunctuations in the composition of a structure due mainly to the random motion of the electron cloud. This is known as thermal noise. For high-precision displacement measurements, thermal noise becomes one of the "fundamental limits to...precision" [7].

In other words, as stated by Andri Gretarsson and Gregory Harry of the Department of Physics at Syracuse University,

> "[Thermal energy] fluctuations are due to coupling between the signal-band degrees of freedom of the detector and the thermal bath of other degrees of freedom."

Because of this coupling, there is dissipation in the system. Luckily, a relationship between thermal noise and dissipation has been found–the Fluctutation-Dissipation Theorem. This theorem claims that

$$x^2(f) = \frac{k_B T}{\pi^2 f^2} Re[Z(f)^{-1}] \tag{2.1}$$

where T is the temperature of the desired object, $Z(f)^{-1}$ is one over the impedance of the excitations of the desired object, f is the frequency, and $x(f)$ is the spectral density of the fluctuations [6]. Then, it has been found that dissipation due only to composition of a material can be approximated as

$$F = -k[1 + i\phi(\omega)]x \tag{2.2}$$

where F is an applied force, k is the spring constant of the material, and x is the displacement of the material [7]. Because of the complex part of this equation, the response lags the force by $\phi(\omega)$; therefore, damping occurs in the system, and by the Fluctuation-Dissipation Theorem, causes thermal noise [7]. As can be seen from Equation 2.2, the larger $\phi(\omega)$ is, the more damping there is in the system. In other words, the smaller that $\phi(\omega)$ is, the better the material can be approximated as a perfect harmonic oscillator, a most desirable property.

Using Equation 2.2, one can see that the equation of motion for such a system is described by

$$m\ddot{x} = -k[1 + i\phi(\omega)]\Delta x + F \tag{2.3}$$

where Δx describes the displacement from equilibrium of the material [7]. From Equation 2.2, it can be shown that

$$Z = im\omega + \frac{k}{i\omega} + \frac{k\phi}{\omega}. \tag{2.4}$$

Therefore, recalling Equation 2.1, since $Z(f)^{-1}$ is proportional to the lag $\phi(\omega)$, in order to reduce the effects of thermal noise, the dissipation due to internal damping must be minimized [6]. It can also be shown that

$$Q = \frac{1}{\phi(\omega_\circ)}[7] \tag{2.5}$$

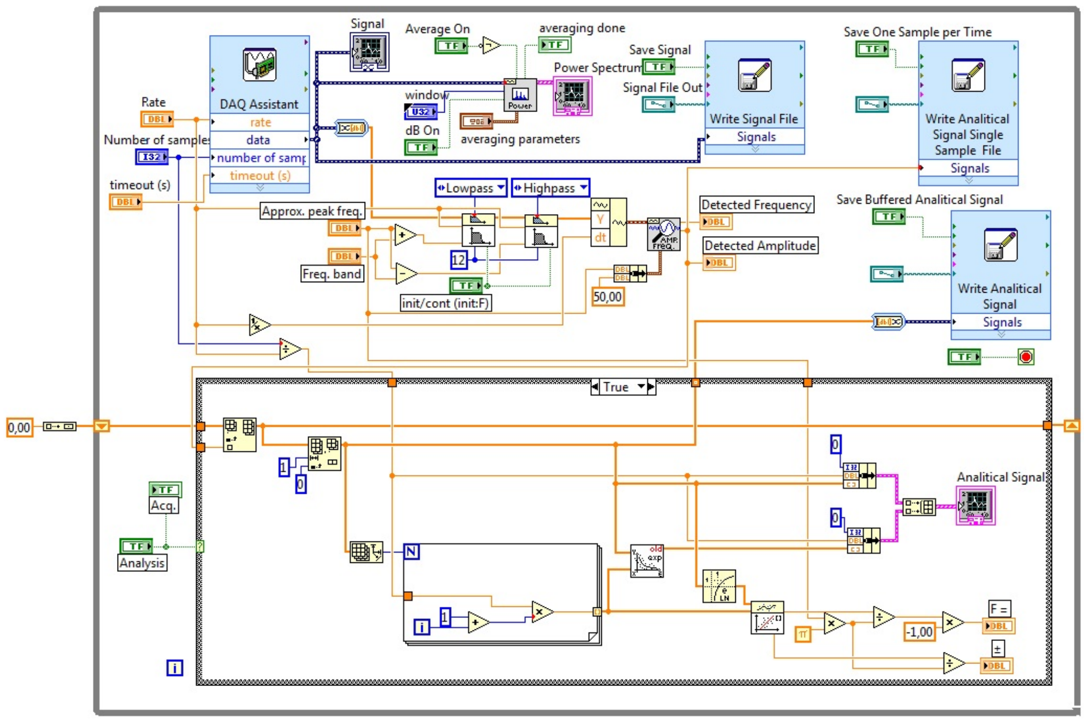
and thus a material with low dissipation is the same as a material with a high quality factor.

Another way of viewing this is to consider another definition of Q. Mainly

$$Q = \frac{\omega_\circ}{\Delta\omega}. \tag{2.6}$$

Then, if Q is very large, this implies that $\Delta\omega$ is very small. In terms of thermal energy, this means that most of the energy is concentrated into a narrow bandwidth of $\Delta\omega$ at resonant frequency and so off resonance the effects due to noise will be less than for a material with a high Q. Therefore, there proves to be great interest in materials with high Q factors. Fused silica proves to be a good candidate.

2.2 Data Acquisition Method



Pictured above is the block diagram code for the LabView program created to analyze the data for the experiments done. The output of the program was to display the signal as determined by the rate and the number of samples as well as to display a Power Spectrum of the incoming signal.

To determine the loss angle, the quality factor was measured using the ring-down time, τ, by fitting the peak-to-peak amplitude using the exponential function.

To actually acquire the data, the fibers were excited to a resonant mode and then allowed to ring-down until the amplitude was about half of the initial amplitude. Measurements were taken as soon as the excitation method was shut off.

2.3 Horizontal Mode Characterization

2.3.1 Experimental Procedure

For this experiment, fused silica fibers with a length of about 0.7 m and diameter of 1.5 mm were used. To suspend the fiber, a steel structure depicted in Figure 2.1 was used. The silica fiber was placed inside a steel box attached to the top of the structure and allowed to hang freely with a load of 5 kg attached to the bottom (the 5 kg mass is added in order to minimize recoil losses). The silica fibers were excited using an electrically conducting piece of metal shaped as a cone placed about 3 cm from the fiber. This cone was then connected to an Agilent 33220A 20 MHz Function/Arbitrary WaveformGenerator and a Matsusada High Voltage Amplifier outside of the vacuum chamber. The sinusoidal input signal produced by the function generator was magnified by 0.92 kV using the voltage amplifier. Knowing that the first resonant mode of the fiber was at about $f_1 = 450$ Hz and that subsequent resonant mode frequencies could be roughly approximated by $f_n = nf_1$, the frequency of the input signal was adjusted until the analytic signal displayed in LabView confirmed resonance (as shown in Figure 2.2).

Figure 2.1: Steel structure used to suspend the fiber

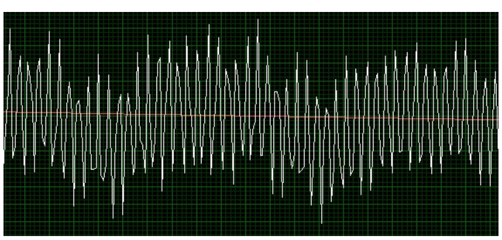


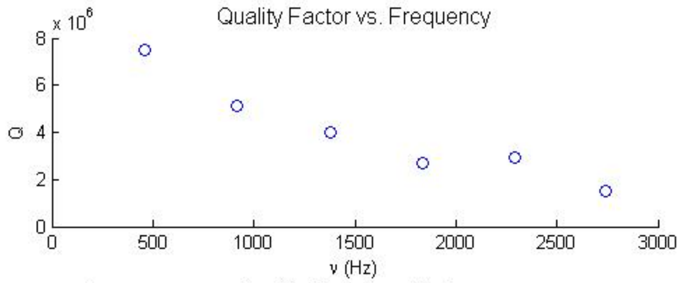
Figure 2.2: Example of analytic signal of the fiber during resonance

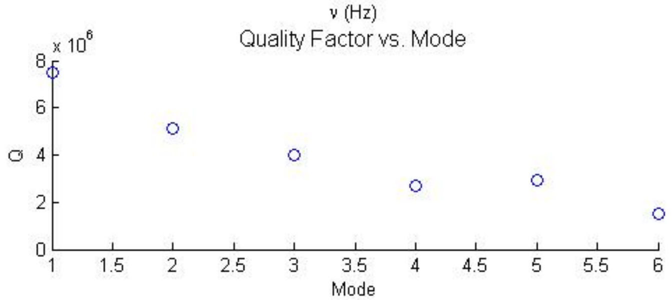
Measurements of the loss angle were made in a vacuum chamber at 2.0x10^{-6} Torr. This was done to minimize losses from collision with gas molecules and the silica fiber. Because the experiment was done in vacuum, all materials inside the chamber had to be cleaned with isopropyl alcohol in order to eliminate residual particles from evaporating in the vacuum and causing losses due to collisions with the fiber.

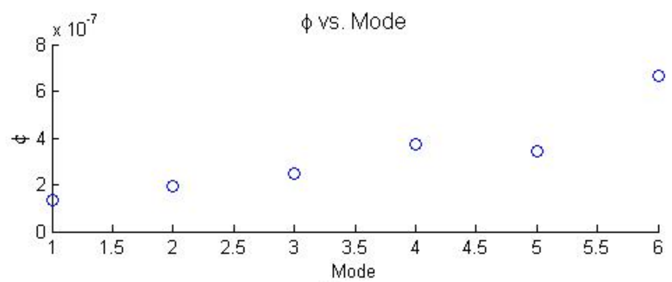
To measure the displacement of the fiber from equilibrium position, two shadow meters were placed in orthogonal orientations with respect to each other. To implement the shadow meters, two photodiodes were used and the meters were positioned such that the fiber dangled in between the photodiodes. With this set up, as soon as the fiber was displaced from equilibrium, whichever photodiode was "covered" by the fiber would output less of a voltage than the "uncovered" photodiode. A power supply voltage of 8.2 V was given, and the shadow meters were fed to a high-pass, low-pass filter and then to the DAQ to be analyzed.

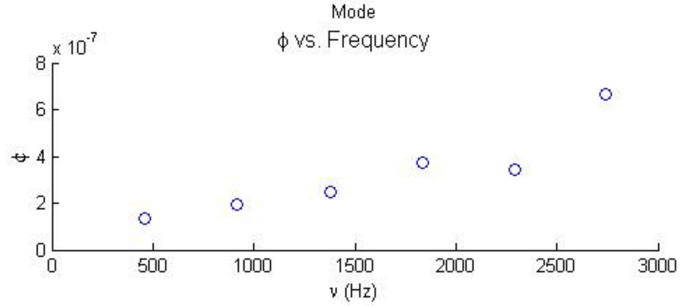
2.3.2 Results

The data collected is summarized in the following graphs and table. Note, due to human error, only one of the two shadow meters was used to collect data.









n	ν (Hz)	Φ
1	459.780	$1.3305\text{x}10^{-7} \pm 6.5213\text{x}10^{-6}$
2	919.323	$1.955\text{x}10^{-7} \pm 8.7526\text{x}10^{-5}$
3	1376.192	$2.4833\text{x}10^{-7} \pm 6.3234\text{x}10^{-5}$
4	1833.3817	$3.7100\text{x}10^{-7} \pm 2.1925\text{x}10^{-5}$
5	2288.8053	$3.4119\text{x}10^{-7} \pm 2.1342\text{x}10^{-5}$
6	2740.8109	$6.67304\text{x}10^{-7} \pm 2.0630\text{x}10^{-5}$

2.3.3 Discussion

From the graphs and data, one can notice a positive correlation between the loss angle and the resonant mode. The fifth mode seems to be a rebel in this case; however, because of the other data points, it can be safely assumed that this is just a statistical error. From a theoretical viewpoint, it is strange to find that the loss angle increases with the resonant mode. In principle, for each resonant mode, the majority of the losses should be due to internal friction–a quality intrinsic to the material itself. With this in mind, one would think that the loss angle would not vary greatly from mode to mode; however, the data speaks differently. This could be evidence of thermal noise due to the standard clamping system used in Virgo+–when the fiber is excited to a higher resonant mode, the recoil losses take on a greater significance. More research would have to confirm this theory.

2.4 Vertical Mode Characterization

2.4.1 Experimental Procedure

The same steel structure used to house the fiber for the violin modes was used again for the bouncing modes. Again, the fiber was placed inside a steel box at the top of the structure and allowed to hang freely. A 5 kg mass was again attached to the base of the fiber for the same reasons stated previously.

In order to excite the vertical modes, magnetic fields were utilized. A solenoid with no core, $r_{inner} = 3$ cm, $r_{outer} = 6$ cm, $R = 26\Omega$, and a height of 6 cm was used. The decision to not use a core in the solenoid arose from the results of the research done at the Instituto Nazionale di Fisica Nucleare in January 1998 that stated "the Q of the pendulum with [an] open solenoid next to it remained the same as that for the pendulum without the solenoid" [8]. In other words, eddy currents introduced from the presence of the solenoid should not have a great impact on the measurements done. The solenoid was then attached to the base of the vacuum chamber as is shown in Figure 2.3.

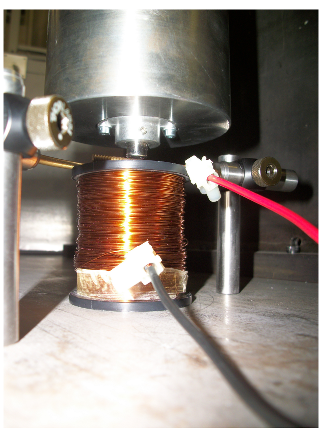


Figure 2.3: The set-up used to excite the bouncing modes

The base of the 5 kg mass was then modified to house a S-10-40-N magnet with a diameter of 10 mm and a height of 40 mm. The magnet stuck out from the mass by about 3 cm. The full set up is shown in Figure 2.4.

The solenoid was then attached to a Kepco BOP 100-4M Bipolar Operational Power Supply and the same function generator used in the horizontal mode excitations.

Figure 2.4: The 5 kg mass with the magnet attached

To actually measure the loss angle of the vertical modes, the shadow meters were utilized once again. While this may seem strange at first (since the shadow meters measure displacements of the fiber in the horizontal direction, not the vertical), this bizarreness proves to be an illusion. While it simplifies the math and the concepts to think of the horizontal and vertical modes acting in orthogonal planes with respect to each other, the reality is that these motions are coupled. Therefore, if one were to excite a resonant bouncing mode, such a state can be detected by measuring the vibrations of the fiber in the horizontal plane. Also, since it is trying to be determined whether or not the coupling between these two oscillations are significant, it only makes sense to do so. From all of this, one can gather that using the shadow meters to measure the loss angle of the vertical modes does not seem like such a bad idea.

Again, the shadow meters were given 8.2 V, fed to a high-pass, low-pass filter, and then to the DAQ for analysis. Measurements were made in the same vacuum at 2.0x10^{-6} Torr and all objects in the vacuum were again cleaned with isopropyl alcohol for the same reasons mentioned earlier.

2.4.2 Results

Only the first mode was excited. It was found that the resonant frequency was $\nu = 5.69$ Hz with $\Phi_x = 3.4339\text{x}10^{-6} \pm 1.7841\text{x}10^{-4}$ and $\Phi_y = 3.4761\text{x}10^{-6} \pm 1.5913\text{x}10^{-3}$.

2.4.3 Discussion

It can be seen from the data that Φ_x and Φ_y are very close in value meaning that the numbers measured can be trusted. From the fact that data was able to be collected at all testifies to the fact that yes, the vertical and horizontal mode coupling proves to be more significant than originally thought. This could potentially bring about some issues since the thermal noise seen by the interferometer will be both from the vertical and horizontal modes when only the horizontal modes were anticipated.

However, it is strange to find that the values found for the verticle loss angle and the horizontal loss angle differ by an order of magnitude. Again, the loss angle should be due mainly to internal friction–a property intrinsic to the material itself. Therefore, there must be other losses in play here. One idea could be that the losses due to the clamping system of the fiber become even more important with the bouncing modes since the clamp was designed to minimize losses only in the horizontal direction. Another source for losses could be the eddy currents discussed earlier although this seems unlikely.

Chapter 3

Shmorgishborg of a Report

3.1 An Explanation

This chapter details other information that I wanted to include in my report but couldn't really find a proper way to do so. I apologize in advance for the mess in organization that this creates, but I couldn't find a better solution. And now, for the seemingly random dosage of information. Please, enjoy!

3.2 Experiment 2: MicroEnergy Harvesting

For this experiment, I worked closely with two graduate students at the University of Perugia: Davide Chiuchiú and Valeria Nico. During this time period, we worked in a clean room analyzing microchips with sixteen membranes on the surface. Chiuchiú had already characterized the microchips by measuring the quality factor of each membrane and the corresponding resonant frequency. While I was at the lab, Nico, Chiuchiú, and I worked on figuring out how to wire all of the membranes together. The goal of this experiment was to see if the microchips could be used to harvest microenergy from the ambient noise that is present for a small, electronic device. The microchip was mounted on a piezo-electric in order to give the excitations, and Nico, Chiuchiú, and I worked to mount the microchip on smaller piezo-electric surfaces in order to mount the system into the electrical device. After this was done, we did tests to determine whether or not grounding all the membranes to the same ground would cause cross-talk between the membranes (and thus cause interference between their respective oscillations). It was found that no such cross-talk exists, and thus the membranes could all share the same ground–a great results for the goal in mind. The next step would be to mount the system into the electronic to see if it could harvest microenergy. Unfortunately, my time at NiPS Lab ended before I could see the end of this experiment.

3.3 Experiment 3: Young's Modulus of Water Glass Bonded Silica Cylinders

This experiment occured near the end of my time at NiPS Lab and proved to be a short experiment. The goal of this experiment was to determine how the concentration of silicate would affect the Young's Modulus of silica bonded together using the water glass technique.

In order to test this, Dr. Flavio Travasso and I bonded several sets of pairs of silica cylinders together using the water glass bonding technique with different concentrations of silicate–6, 10, and 20 percent concentrations. The pairs of cylinders where then allowed to sit for one week with 250 g of weight on each pair in order to ensure proper bonding. Then, we used a material testing machine to gather data about the cylinders to determine the Young's Modulus. Several pairs for each concentration were made, and each pair was pulled with the machine until it broke.

This experiment was done to completion; however, I felt as though the main experiment discussed in the previous chapter deserved a full treatment since it occupied the majority of my time in Italy.

3.4 Summer School: Energy Harvesting at Micro and Nanoscales

While at the NiPS Lab in Perugia, Italy, I was given the opportunity to attend the third edition of the Summer School that NiPS hosts. This summer, the school met at the Ettore Majorana Center–a magnificent scientific center rich in history–in Erice, Sicily to discuss energy harvesting at very small scales. During my week spent at the school, I attended lectures ranging in topics from the thermoelectric effect to quantum energy harvesting and learned about microenergy from scientists at the top of this field. Beyond this, I was able to discuss with other researchers the current work they are conducting in this field through poster sessions and workshops held at the summer school.

In short, the summer school allowed me to interact with professionals and researchers in this field on a more personal level while simultaneously learning more in depth about energy harvesting. This was an amazing opportunity that I will not forget, and the discussions about microenergies has definitely piqued my curiousity. (On a fun side note, I will never forget the gala dinner for I was chosen by Markus Buttiker to give the final speech due to the fact that I was the youngest person attending the school. What a night to remember!)

3.5 Acknowledgements

I would like to thank Dr. Flavio Travasso, Dr. Helios Vocca, Dr. Luca Gammaitoni, and the entire NiPS Laboratory for welcoming me into the lab and putting up with me all summer. Also, I must extend my thanks to Dr. Bernard Whiting, Kristen Nichola, Dr. Guido Mueller, and Antonis Mytidis for putting together this entire program and giving students the opportunity to experience gravitational wave research first-hand. Thanks is also due to the University of Perugia for use of their facilities and equipment. Last, but certainly not least, I would like to thank the National Science Foundation and University of Florida for their financial support and accomadations.

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