Ground Testing for LISA Test Masses with a Torsion Pendulum

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Abstract:

One of the most important aspects to the LISA mission is the ability to put the Test Mass in drag free motion. In order to achieve this pure free fall motion, capacitive sensors will measure the position of the test mass relative to the housing. Many stray forces can cause accelerations on the test mass, such as cosmic ray charging, thermal gradients, and the electrostatic coupling from building the sensors around the test mass. This paper discusses some measurements and possible solutions to these stray forces.

The Laser Interferometer Space Antenna (LISA) will consist of 3 satellites in triangular configuration orbiting the sun. [1] The purpose of these spacecraft is to detect the gravitational wave strain of binaries with frequencies from 3 mHz to 0.1 Hz. This wave strain will expand and contract the distances of the test masses within each spacecraft. [2]Each satellite will have two test masses, each being the end mirror for an interferometer. These masses will configured at an angle of 60 degrees from each other and linked to another mass 5 million km away. LISA will use a system of lasers to create an interferometer, measuring the distances between the two masses facing each other. For gravity to be the only force affecting these masses, they must be in perfect free fall, which will be provided by the spacecrafts μN thrusters' ability to keep the test mass centered inside the housing along the interferometer axis. [3] "Achieving the LISA gravitational sensitivity requires the test masses are in free fall with residual accelerations below 3 $fm/s^2/\sqrt{Hz}$." [4]In order to accomplish this task, "capacitive sensors will provide a readout of the relative position of the satellite to the freely flying test masses." One obstacle is being able to deal with forces that can affect this free fall. This paper will name some of those disturbances as well as discuss how to measure and compensate for coupling of the mass to the satellite, charges on the test mass and a change in temperature of the system.

The test mass for the LISA mission will be a gold/platinum cube, 46 mm on each side, and 2 kg in mass. The torsion pendulum test mass is only for ground testing. The torsion pendulum test mass is gold coated aluminum of approximately 100g. This mass is mounted on a torsion pendulum with a 1 m in length and 35 μm in thickness fused silica fiber. Within the 3 dimensions of the cube, capacitive sensors measure the relative position to the housing. [5]Surrounding the test mass will be the electrostatic position sensors or a gravitational reference sensor (GRS) to measure the difference in capacitance between the test mass and opposing pairs of electrodes, which will give the distance since capacitance is inversely proportional to the distance. Using these sensors both the translational and rotational motion can be measured by utilizing two electrodes on each face. For this measurement, a 100 kHz frequency is pumped into the injection electrodes shown in Fig 1. This produces a measurable current that is sent to a transformer. One can see by the diagram that if the capacitance is the same, then the incoming currents will also be the same and cancel each other.

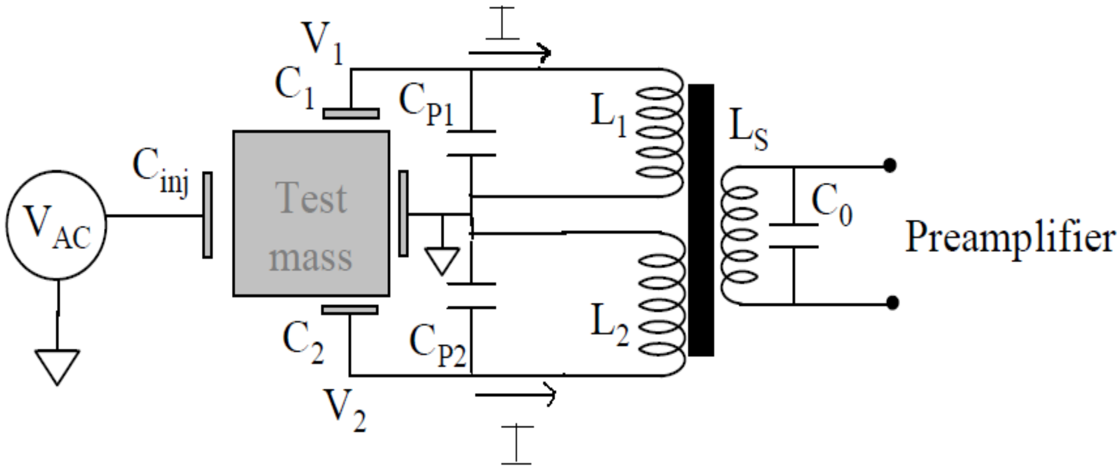

Fig 1. 100 kHz voltage applied to the test mass. A change in capacitance occurs when the test mass moves. The current is measured from each side across the transformer.

To measure translational motion, the position of the two sensors on the face in which the dimension of motion is taking place must be summed. The difference of these two capacitors sensors under translational motion only will read 0. Under rotational motion of the test mass, the difference of the sensors on the same face will give a measurement of rotation, while the sum would read 0.

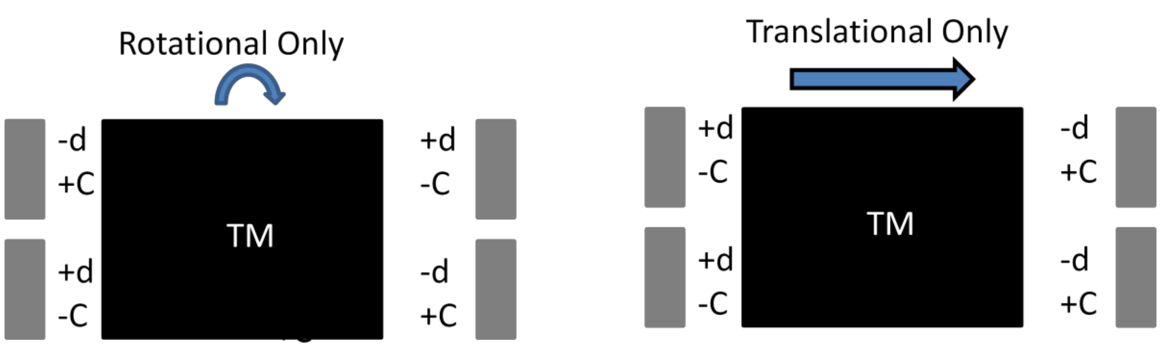

Fig 2. Capacitive sensors change in capacitance as the distance changes. This measures the movement of the pendulum.

These angle measurements are crucial in the measuring of the torque. In space, for the LISA and LISA Pathfinder, there are no torques to be measured. In flight, forces change the distance between the free falling test mass and the sensors. Most sources of force noise also produce torque noise in the torsion pendulum. Force noise can be measured in terms of torque during torsion pendulum measurements. It must also be noted that there is also an autocollimator readout to measure the torsion angle. The angle, φ, is measured overtime and applied to the following equation:

$$\frac{Id^2\varphi}{dt^2} + \frac{bd\varphi}{dt} + c\varphi = N(t) \qquad (1)^6$$

Where I is the moment of inertia of the cube, b is the damping coefficient and c is the restoring torque constant. The measurement of the angle, along with its derivatives, will give the torque being applied on the pendulum.

Before diving deeper into torque measurement, it may be beneficial to explain the source of these torques being applied to the system. [4]There are many ways in which "DC electric fields can combine with test mass charging and thermal dielectric voltage noise to create significant force noise." The arrival of random cosmic rays induces charge on the test mass. These changes arrive in a random walk process, but are still statistical in nature. Also, creating a gold/platinum alloy creates different work functions in the metal. [4]Cutting metals also creates different crystal faces with different work functions, but this effect is small and not likely to be a significant problem. Thermal gradients that appear across electrodes can cause noise as a radiometer effect. Under low pressure circumstances, higher temperatures encourage movement of molecules that in turn transfer momentum to the test mass. Radiation pressure, photons, can also transfer momentum.

There is also the issue of weak coupling ($k=m\omega_p^2$), within the system. Internal forces from building something around the test mass create coupling 'stiffness' with a very weak spring constant. The μN thrusters on the LISA satellites will be used to create a free falling environment, but will also create these disturbances. [6]The relative motion of the satellite will create a coupling, and "the most significant source of spring like coupling (is) the AC voltage bias used for capacitive position readout." Physical attributes of the pendulum can also cause disturbances. The surface of the pendulum could become contaminated before set in a vacuum changing the work functions and magnetic properties of the mass. Also, outgassing—the slow release of molecules trapped inside the mass—could contaminate the vacuum and interfere with measurements. All of these problems will potentially create noise in the system measurements and learning to deal with these is crucial to LISA success.

If the noise comes in very fast with a high frequency, it does not cause a problem because it can be easily averaged out. If the noise is of a very low frequency, it will also have little effect. We are interested in the amplitude of an AC signal, which can still be seen well under slow noises. LISA is dominated by forces in its noise budget from 0.1 mHz to 3 mHz, but we are interested in all sensitivity levels from 0.1mHz to 0.1Hz. When the noise is at the frequency we want to measure, an error occurs. Though the noise follow a random walk process, they still obey statistics, so an expectation value can be found. Thus we may take the power spectral density to capture the frequency of this stochastic process. We can use the Fast Fourrier Transform (FFT) of the autocorrelation function to separate the noise and identify the signal we wish to measure. Using this method, most noise can be subtracted from the signal.

Before moving to the next measurement, it is necessary to express torque electrostatically. In the system the difference of potentials is important. Thus the sum of charges can be expressed as:

$$\sum_i q_i = \sum_i C_i (V_i - V_{TM}) \quad (2)$$

Allowing the sum to be expanded this equation simplifies.

$$Q = \sum_i C_i V_i - V_{TM} \sum_i C_i \quad (3)$$

Solving for the voltage on the test mass gives:

$$V_{TM} = \frac{Q}{C_{TOT}} + \frac{\sum_i C_i V_i}{C_{TOT}} \quad (4)$$

Using the equation for potential energy and taking the derivative to find the force yields the following equations:

$$U = \frac{1}{2} \sum_i C_i \, (V_i - V_{TM})^2 \quad (5)$$

$$F_x = -\frac{1}{2} \sum_i \frac{\partial C_i}{\partial x} (V_i - V_{TM})^2 \quad (6)$$

Or in terms of torque and angle φ:

$$N_\varphi = -\frac{1}{2} \sum_i \frac{\partial C_i}{\partial \varphi} (V_i - V_{TM})^2 \quad (7)$$

One of the problems mentioned earlier was the charging of the test mass from cosmic sources. In order to deal with these charges there must be a way to measure the charge on the test mass. Plugging (5) into (7) yields:

$$F_x = -\sum_i \frac{\partial C_i}{\partial x} (V_i - \frac{Q}{C_{TOT}} - \frac{\sum_i C_i V_i}{C_{TOT}})^2 \quad (8)$$

In order to know the force on the pendulum, we must know the charge on the mass and the difference of voltages, but measuring the charge is no direct task. If the voltage were to oscillate at some chosen frequency, then the value of the charge can be measured, and therefore the force can be calculated.

In the experimental setup, a modulation voltage (V_{mod}) of 9.5V is applied on the four Z electrodes at 3 mHz shown in Fig 3. This method simulates a charge on the test mass by creating an electric field and a potential difference between the mass and the surrounding electrode housing. The x electrode senses a buildup of charge.

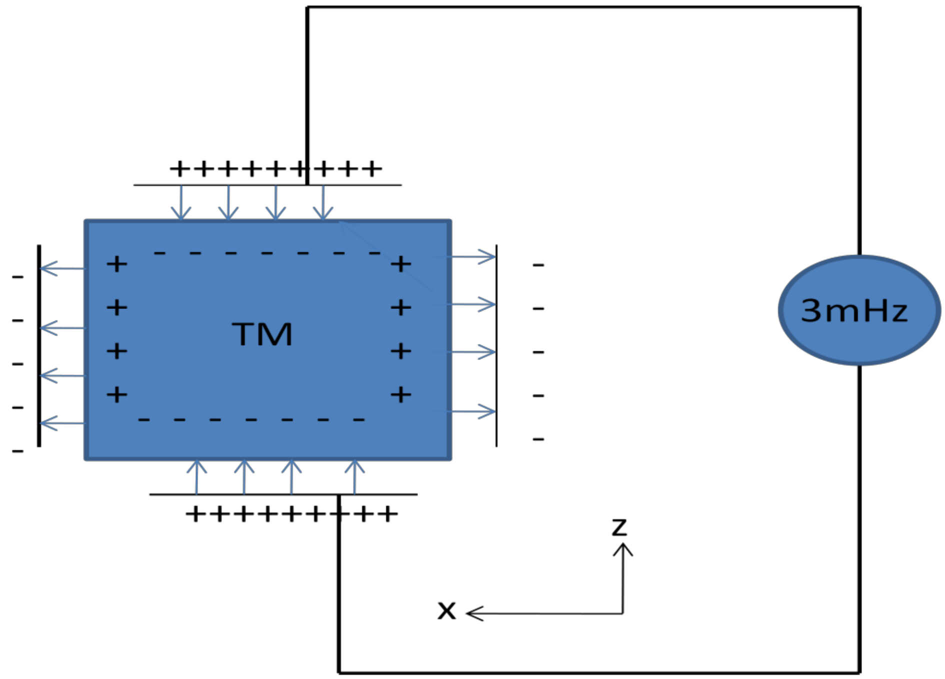

Fig 3: A 3 mHz modulation voltage induces a charge on the test mass.

In experiment, this changes our equation for torque. Summing the term out for V_{TM} over the 3 axis yields:

$$V_{TM} = \frac{Q}{C_{TOT}} + \frac{\sum_x C_x V_x}{C_{TOT}} + \frac{\sum_y C_y V_y}{C_{TOT}} + \frac{\sum_z C_z V_{z0}}{C_{TOT}} + \frac{\sum_z C_z V_{mod} \sin(\omega t)}{C_{TOT}} \quad (9)$$

For easier computation the DC component will be known as:

$$V_{TM0} \equiv \frac{Q}{C_{TOT}} + \frac{\sum_i C_i V_i}{C_{TOT}} \quad (10)$$

The AC component will simply be summed over its 4 electrodes to give:

$$\frac{\sum_z C_z V_{mod} \sin(\omega t)}{C_{TOT}} = \frac{4 C_z V_{mod} \sin(\omega t)}{C_{TOT}} \quad (11)$$

Now the torque can be expressed as:

$$N_\varphi = -\frac{1}{2} \sum_i \frac{\partial C_i}{\partial \varphi} \left(V_i - V_{TM0} - \frac{4 C_z V_{mod} \sin(\omega t)}{C_{TOT}}\right)^2 \quad (12)$$

$$N_\varphi = -\frac{1}{2} \sum_i \frac{\partial C_i}{\partial \varphi} \left(V_i^2 + V_{TM0}^2 + \frac{16 V_{mod}^2 C_z^2 \sin(\omega t)^2}{C_{TOT}^2} - 2 V_i V_{TM0} - \frac{8 V_i C_z V_{mod} \sin(\omega t)}{C_{TOT}} + \frac{8 V_{TM0} C_z V_{mod} \sin(\omega t)}{C_{TOT}}\right) \quad (13)$$

$$N_\varphi = -\frac{1}{2} \sum_i \frac{\partial C_i}{\partial \varphi} \left(V_i^2 + V_{TM0}^2 + \frac{8 V_{mod}^2 C_z^2}{C_{TOT}^2} + \frac{8 V_{mod}^2 C_z^2 \cos(2\omega t)}{C_{TOT}^2} - 2 V_i V_{TM0} - \frac{8 C_z V_{mod} \sin(\omega t)}{C_{TOT}} (V_i - V_{TM0})\right) \quad (14)$$

Now, this can be separated into a DC component and parts dependent on 1ω and 2ω.

$$N_{1\omega} = \frac{4 C_z V_{mod} \sin(\omega t)}{C_{TOT}} \left(\frac{Q}{C_{TOT}} + \frac{\sum_i C_i V_i}{C_{TOT}}\right) \sum_i \frac{\partial C_i}{\partial \varphi} + \sum_i \frac{\partial C_i}{\partial \varphi} V_i \quad (15)$$

When the test mass is centered $\sum_i \frac{\partial C_i}{\partial \varphi}$ is equal to zero.

$$N_{2\omega} = \frac{4 V_{mod}^2 C_z^2 \cos(2\omega t)}{C_{TOT}^2} \sum_i \frac{\partial C_i}{\partial \varphi} \quad (16)$$

Other measurements can be made to help identify and reduce disturbances in the test masses. By moving the sensor back and forth in the X, Y plane and rotationally about φ, the torque by the coupling effect can be calculated. During this measurement a motor moves the sensor in a certain direction, and the position of the pendulum is recorded (using the same position/rotation measurement in equation 1). The signal amplitude of the measured angle is broken into sine and cosine components for both the 1ω and 2ω signal of the test mass modulation voltage. Labview was used to run the program of moving the motors and Matlab was used to analyze the data.

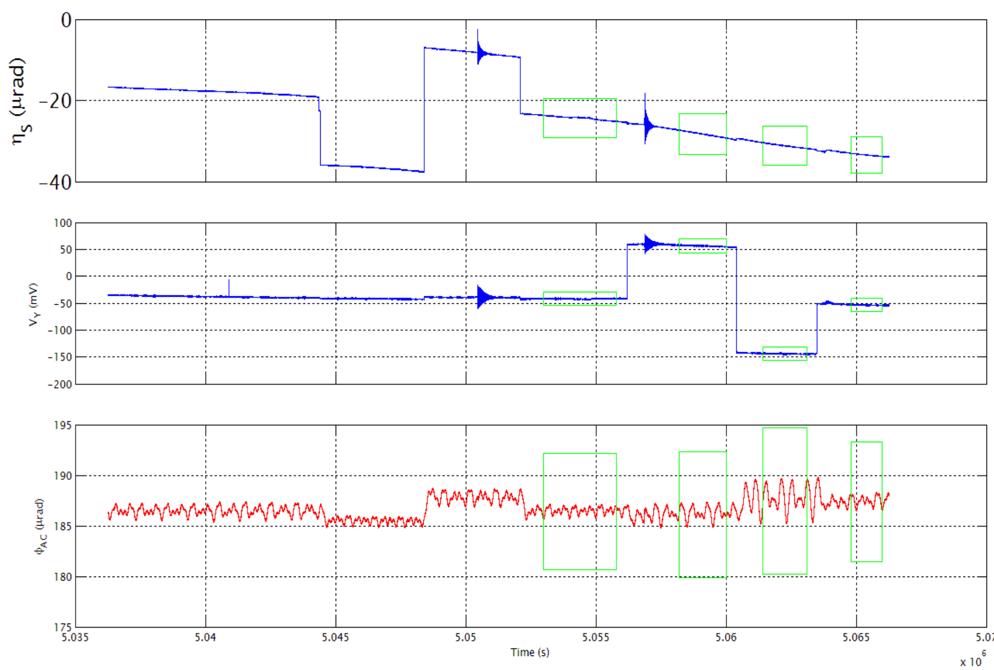

Fig 4: The motor scan can be seen clearly in the Y direction (denoted Vy). The X direction (η) and φ are also shown.

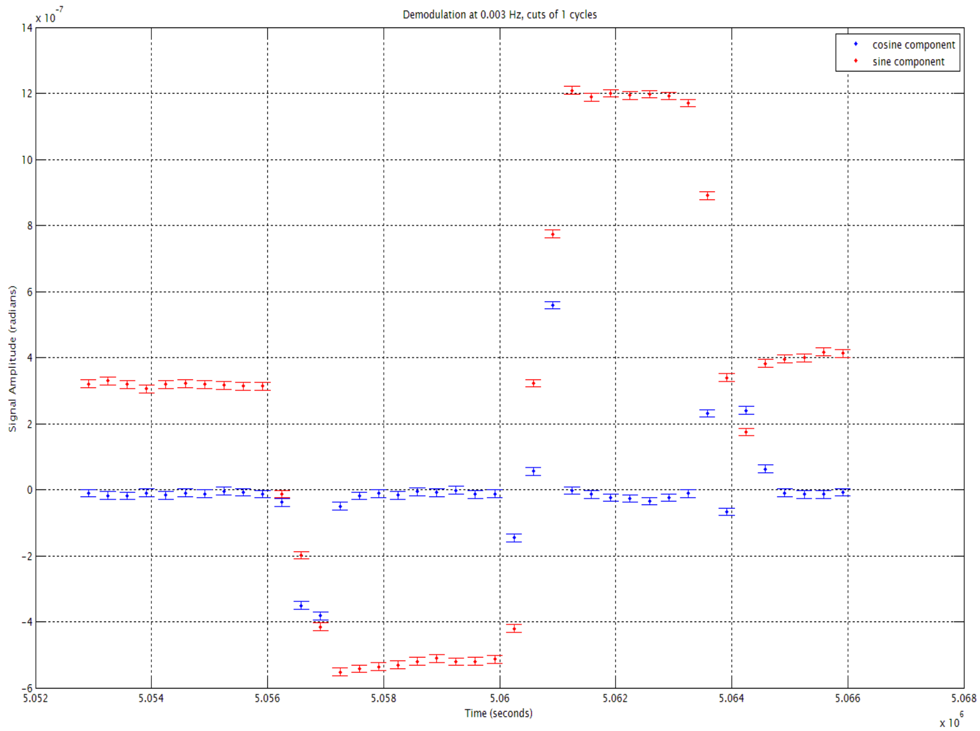

Fig 5: Sine and Cosine components of signal amplitude at 1 ω during a motor scan in the Y direction.
Using equation 1 this signal amplitude can be converted into a torque in terms of sines and cosines that is measured by taking the first and second derivative of the data points measured.

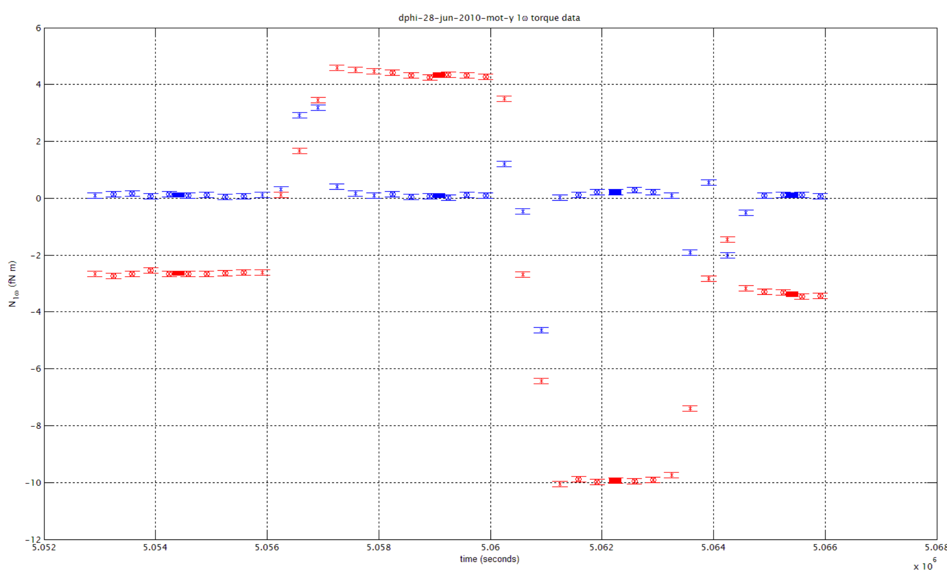

Fig 6: Components of sine and cosine torque of the motor scan in the Y direction.

With these measurements, the dependence of torque on the angle of motion can be calculated as seen here by the sine component of torque vs. movement in the Y direction.

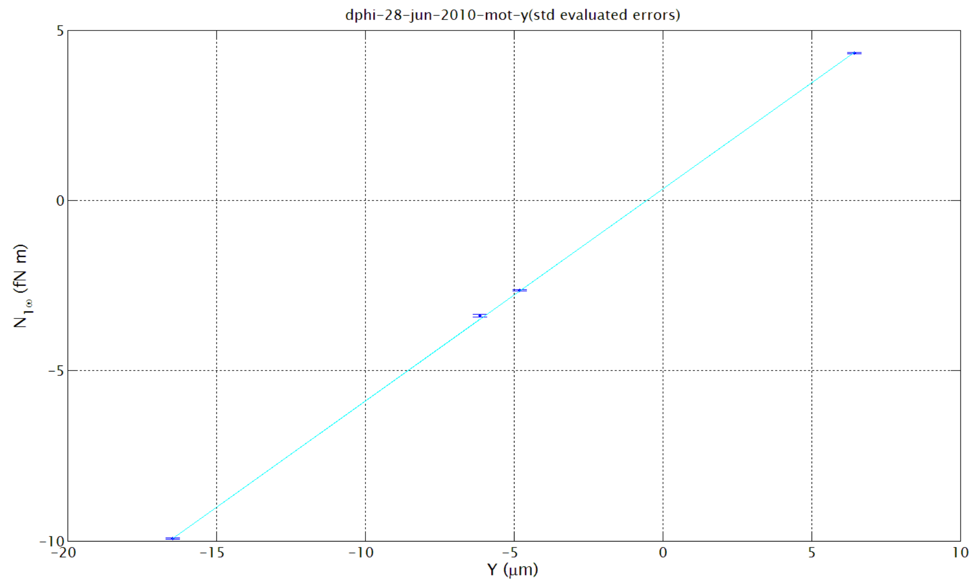

Fig 7: Torque as a function of distance in Y. Shows the slope as the coupling coefficient

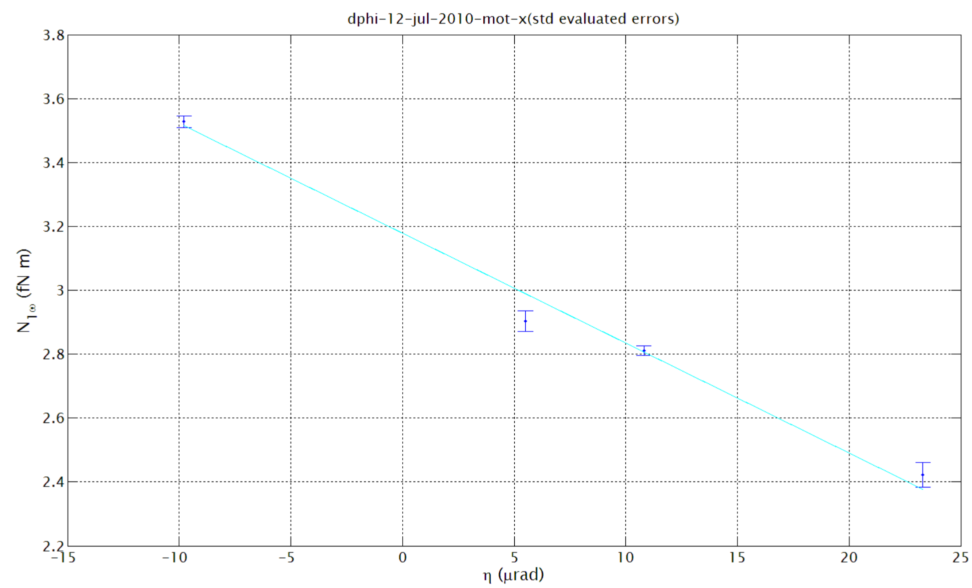

Fig 8: The coupling coefficient for an X motor scan shown at 1ω.

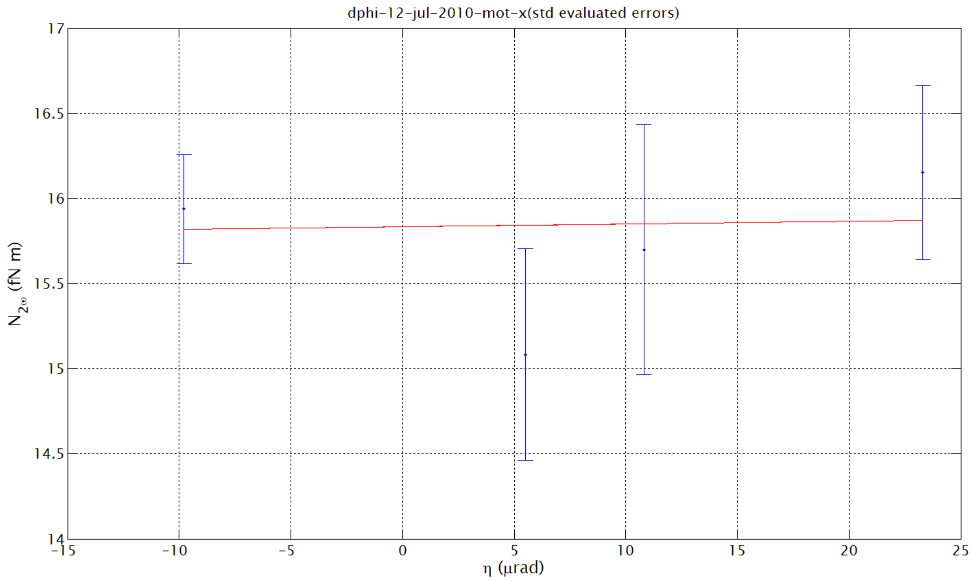

Fig 9: The coupling coefficient for X motor scan appears to be nearly 0. Same applies for Y motor scan.

Expanding the force out as a sum one can write the force as

$$F = F_0 + \frac{\partial F}{\partial x} x \qquad (17)$$

Where $\frac{\partial F}{\partial x}$ is the spring constant calculated through this measurement. In the case of the torsion pendulum, $\frac{\partial N}{\partial x}$ *or* $\frac{\partial N}{\partial y}$ are relative to 1ω and 2ω frequencies of the modulation voltage.

Equations 15 and 16 show why the X and Y motor scans will only give dependencies at 1ω and φ motor scans have dependency on both 1ω and 2ω. All terms in $N_{2\omega}$ are dependent on $\frac{\partial C_i}{\partial \varphi}$ which is changed drastically in the φ scan. X and Y motor scans show dependencies at 1ω because they have terms that do not depend on $\frac{\partial C_i}{\partial \varphi}$. This dependency can be seen in Fig 11.

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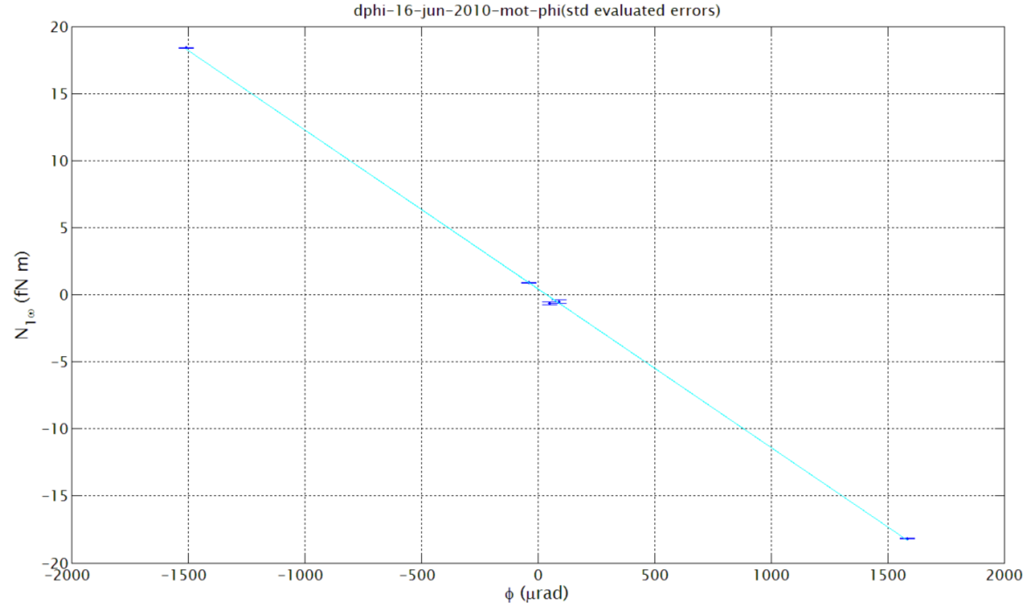

Fig 10: φ motor scan. Correlation coefficients at 1ω frequency.

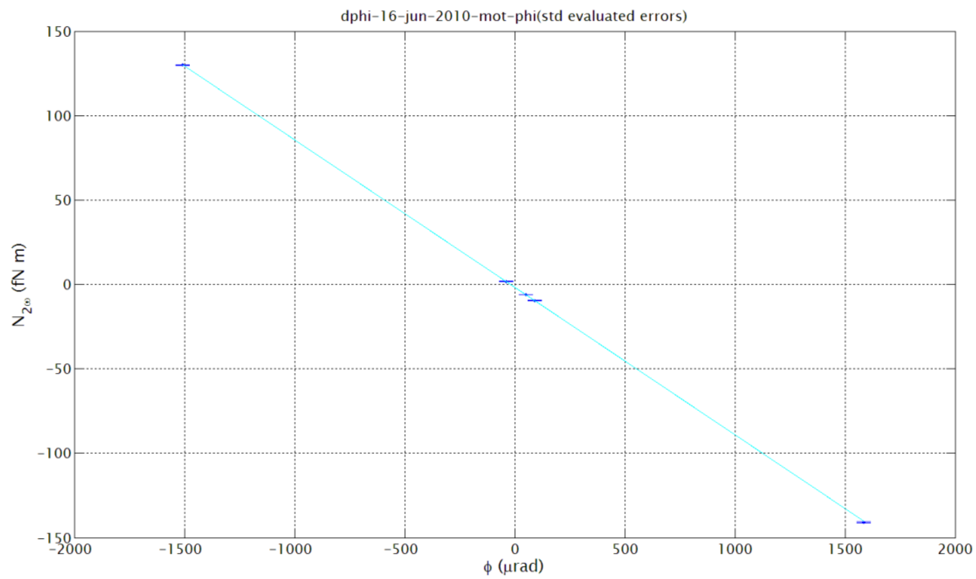

Fig 11: φ Dependency shown on 2ω torque.

$$\frac{\partial N}{\partial q} = \frac{1}{C_{TOT}}\left(-\sum_i \frac{\partial C_i}{\partial \varphi} V_i + \left(\sum_i \frac{\partial C_i}{\partial \varphi}\left(\frac{Q}{C_{TOT}} - \frac{\sum_i C_i V_i}{C_{TOT}}\right)\right)\right) \tag{18}$$

The measured 1ω signal is proportional to the coupling charge ($\frac{\partial N}{\partial q}$). The torque caused by the charge is what we want to null. In the LISA mission ($\frac{\partial F}{\partial q}$), keeping the test mass centered, is crucial for drag free motion. By using these equations, torque's sensitivity on charge (or on test mass potential) can be found. In order to remove any torque on the test mass, a compensation voltage is applied to the four X electrodes. When the test mass is centered, the second term in equation 18 disappears. This is due to the

$\sum_i \frac{\partial C_i}{\partial \varphi}$ being zero when centered. Even if the test mass is not centered, the second term is dependent on 2ω, which when made very small, causes this term to fall away. In turn the equation is simplified to:

$$\frac{\partial N}{\partial q} = -\Delta\varphi \left|\frac{\partial C_i}{\partial \varphi}\right| \tag{19}$$

Where $\Delta\varphi \equiv \frac{1}{\left|\frac{\partial C_i}{\partial \varphi}\right|} \sum_i \frac{\partial C_i}{\partial \varphi} \delta V_i$.

The results of the motor scans can be seen in the measuring of Δφ. The importance of measuring the Δφ is to be able to null the torque on the pendulum. This is discussed mathematically in the paper. For a motor scan done on the 12th of July dN/dη was found to be -38.61 pN and dN/dY was 941.97 pN which are quite different from their values found on May 27th of -61pN and 479 pN respectively. This means that the spatial variation in the electrostatic field, that produces these dependencies, changed overtime. Looking at the Δφ information with the old dependency measurement it is clearly noticeable that the motion in Y is not subtracted from the Delta phi data. The new values for the correction coefficients look much better.

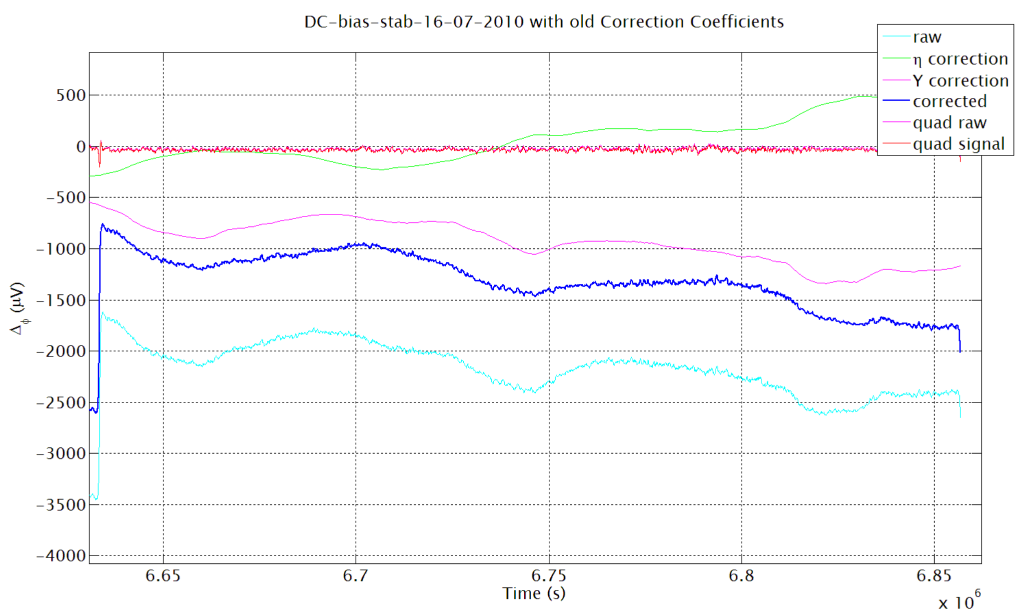

Fig 12: With the old correction data, the corrected data seems dependent on the Y correction.

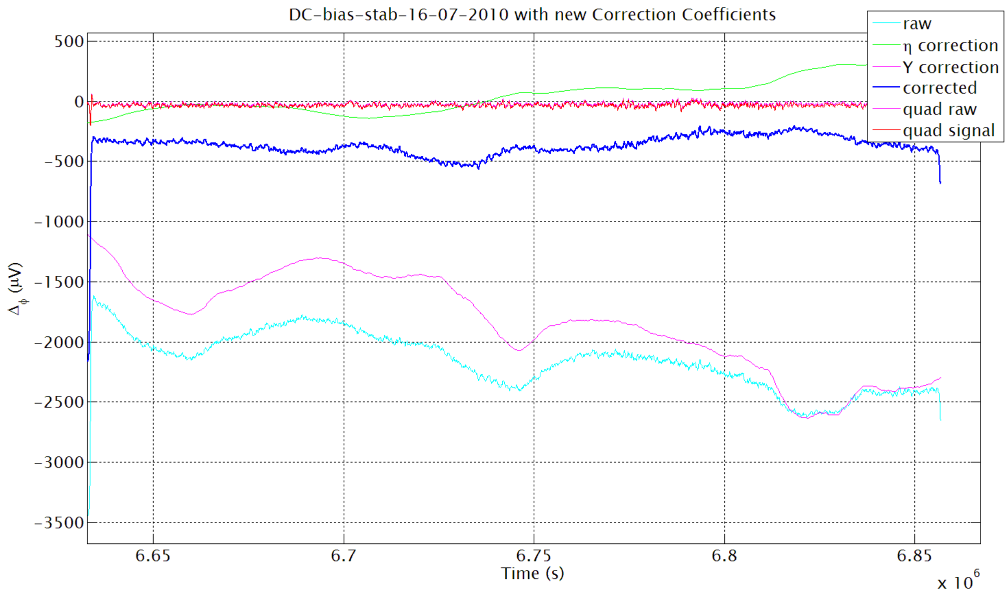

Fig 13: The same data with the new correction coefficient shows the Y correction data being subtracted from the Corrected data.

A compensation on the four X electrodes voltage is used to null the $\Delta\varphi$, and in turn, diminish the torque on the pendulum. Using a program in Labview this compensation voltage is changed over time. This change in DC bias reveals the compensation voltage that best removes this $\Delta\varphi$. An example of a DC bias scan shown in Fig 14 shows the compensation voltage going from -8mV to 12 mV at 4 mV steps of 4,500 seconds each. This scan reveals that the ideal compensation voltage is -10.43 mV. The compensation voltage being applied at the time of the scan gave a sum of -10.95 mV which gives us a difference of about half a mV. We also found that by only modulating with the top or bottom electrodes, instead of all four, will change the value of the ideal compensation voltage that nulls the 1ω term.

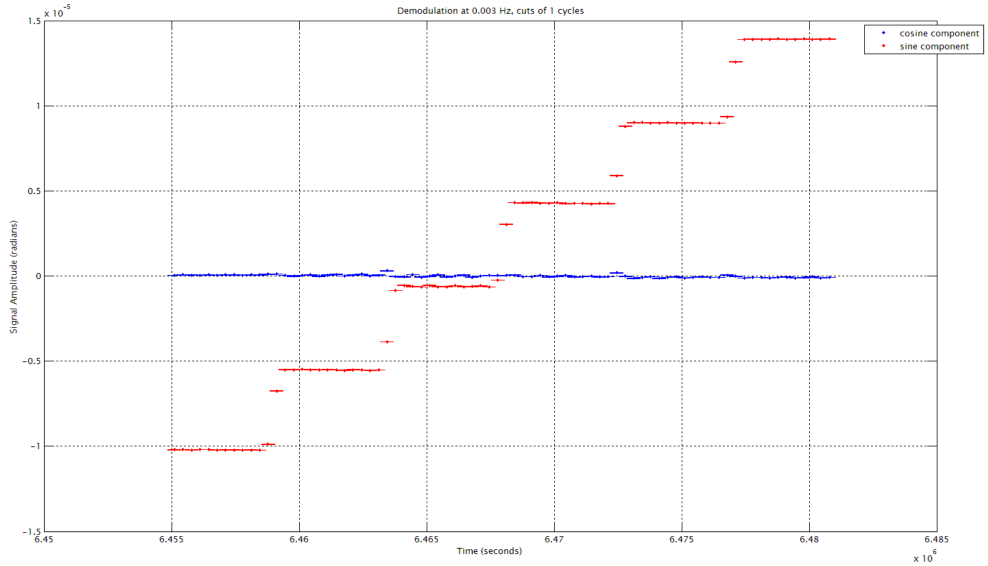

Fig 14: Signal amplitude changing as the compensation voltage changes.

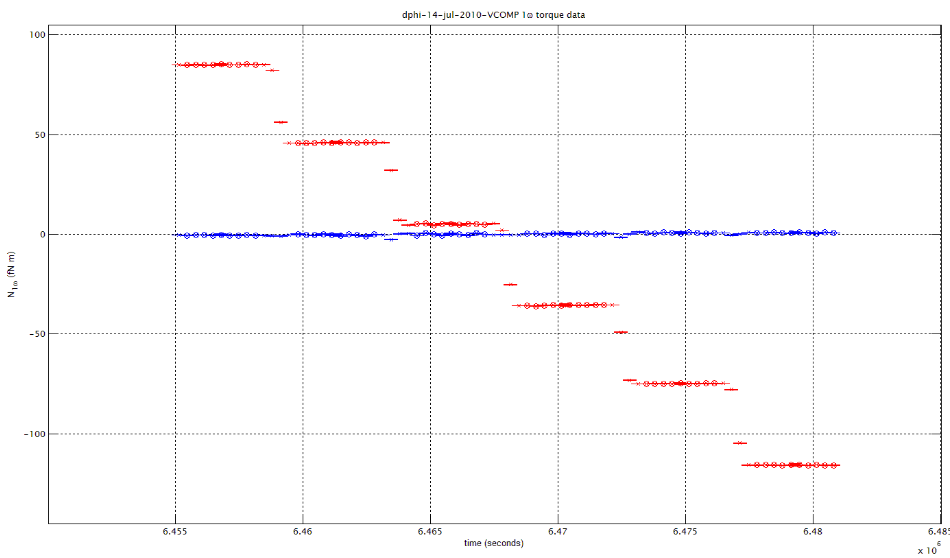

Fig 15: Torque on the pendulum during the compensation voltage scan.

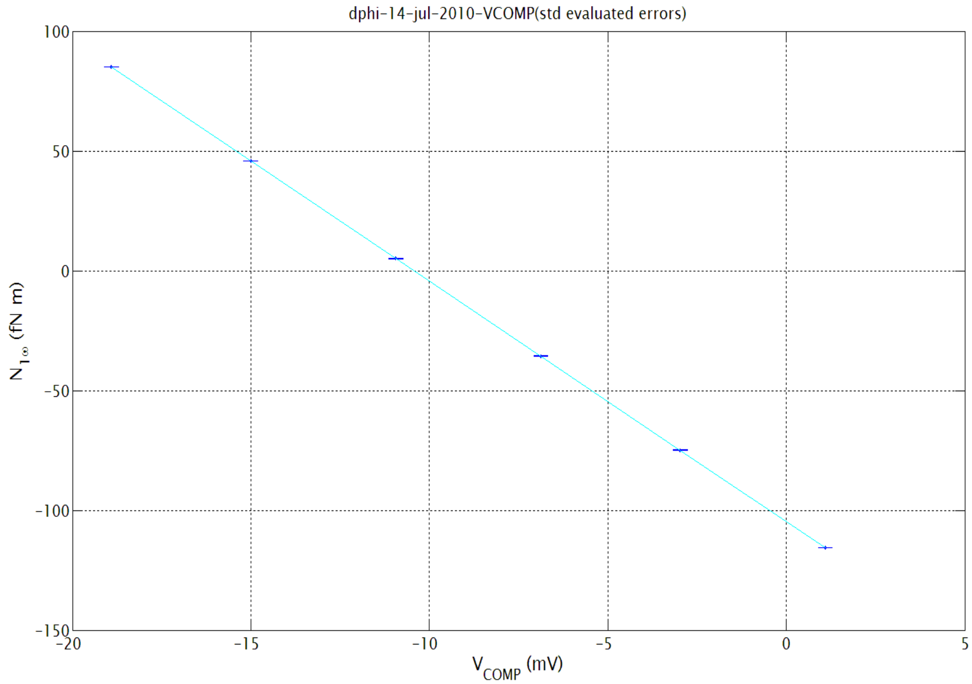

Fig 16: Torque versus compensation Voltage.

Given values for Vcomp of -6.4 mV and 7.6 mV for using the East and West faces of X face respectively, the compensation voltage is missed by .6mV. This slope is calculable from the principles of equation 15.

These are good measurements considering the compensation voltage could have underlying drift of more than .5mV over the 9+ hours of data collection for these experiments. Beforehand, the data was not being digitized correctly. The digital to analog converter has 16 bit resolution from -10.24 to 10.24, so the compensation voltage was rounded to 313μV. In order to solve this problem a code was written to digitize the compensation separately for each voltage applied. This resulted in a much better linear fit.

Another measurement included varying the temperature within the test mass housing. The Δφ data was also measured over time to monitor possible fluctuations. A program was created in Matlab to plot the Δφ time series data and the φ data time series with the temperature data. Measurements were taken under different temperatures and changing temperature. While the temperature and Δφ have no obvious correlation, the φ time series and temperature have a noticeable dependence. This can be seen in Figure 16 as the temperature is dropped from 30° C to 25° C.

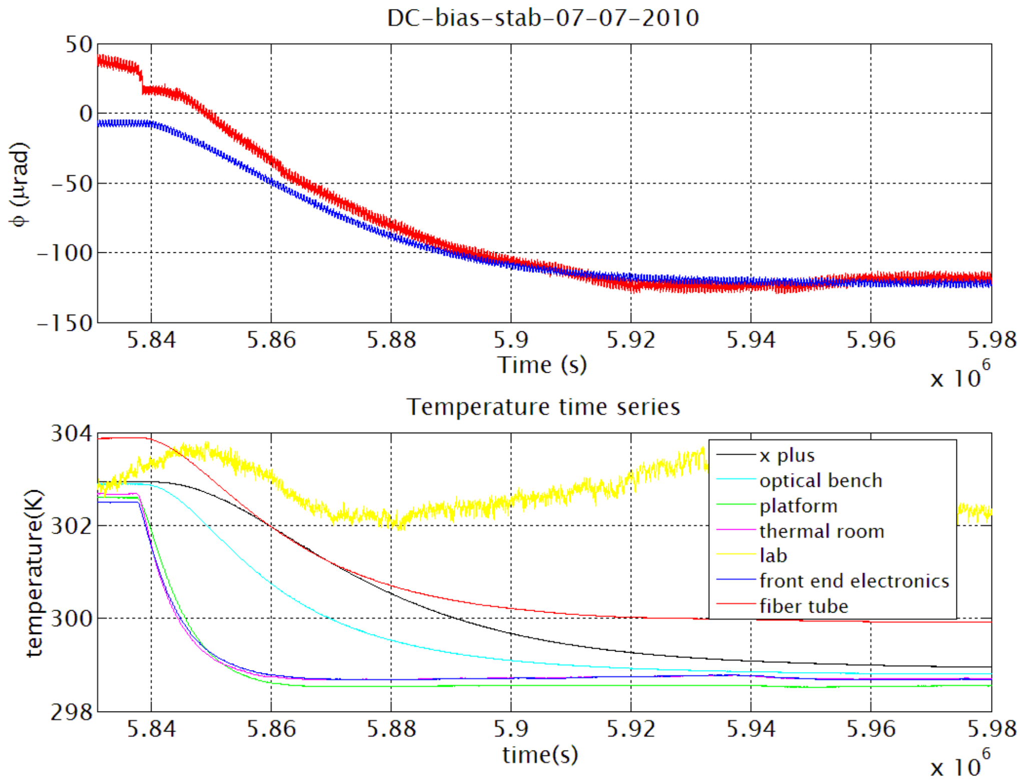

Fig 17: φ time series data ploted with the temperature time series data.

In order to analyze this data, a program was written to interpolate the temperature data in the same time series as the phi data. Then a least square fit was applied in terms of temperature.

$$\varphi = \varphi_0 + \frac{\partial \varphi}{\partial T}(T - \langle T \rangle) \qquad (20)$$

Where $\frac{\partial \varphi}{\partial t}$was nearly always 30µRad $\pm$3µRad

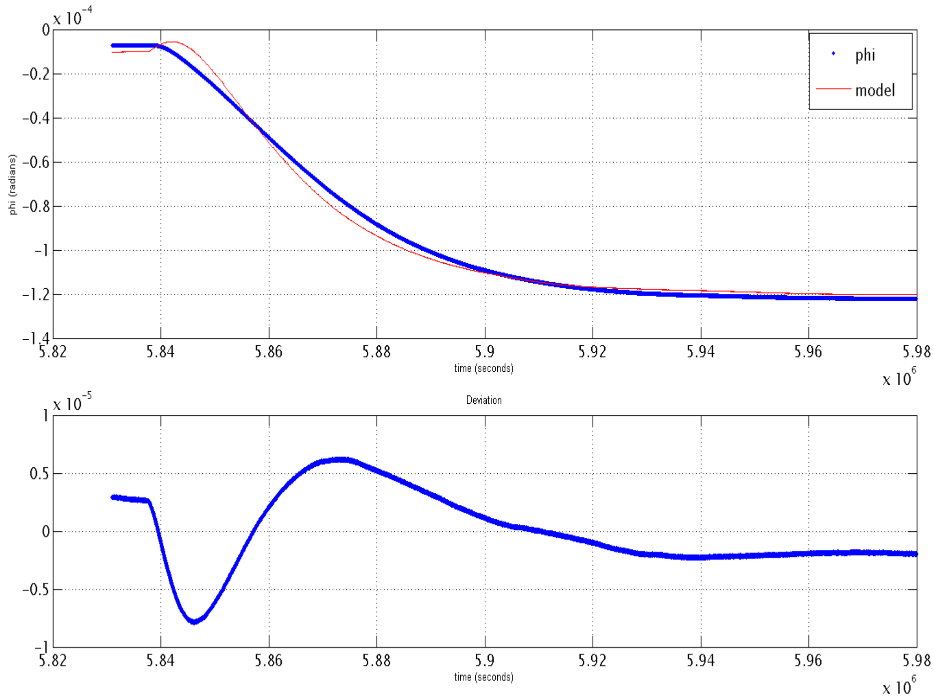

Fig 18: φ data interpolated into the same time series as change in temperature. The bottom graph is the deviation of this interpolation model from the actual φ data.

The data collection for this measurement also possessed a fundamental flaw. As the temperature changed in the housing, it caused a translation on the test mass. Also, a change in the temperature changed the frequency that gave the best Q factor for the modulation.

In conclusion, there are many noise sources that yield a DC bias on the test mass. Ground based testing at the University of Trento will help prepare LISA and LISA Pathfinder. By taking measurements of these noise sources, the electrostatic and magnetic properties of the test mass. The sources of force noise can be measured in different ways. We can deal with electrostatic coupling by moving the test mass and finding the coupling coefficients of the test mass with the sensors. The forces caused by the charging up of the test mass by cosmic sources can be removed by making the Δφ go to zero by an applied compensation voltage. The temperature was also shown to affect the angle φ of the pendulum.

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