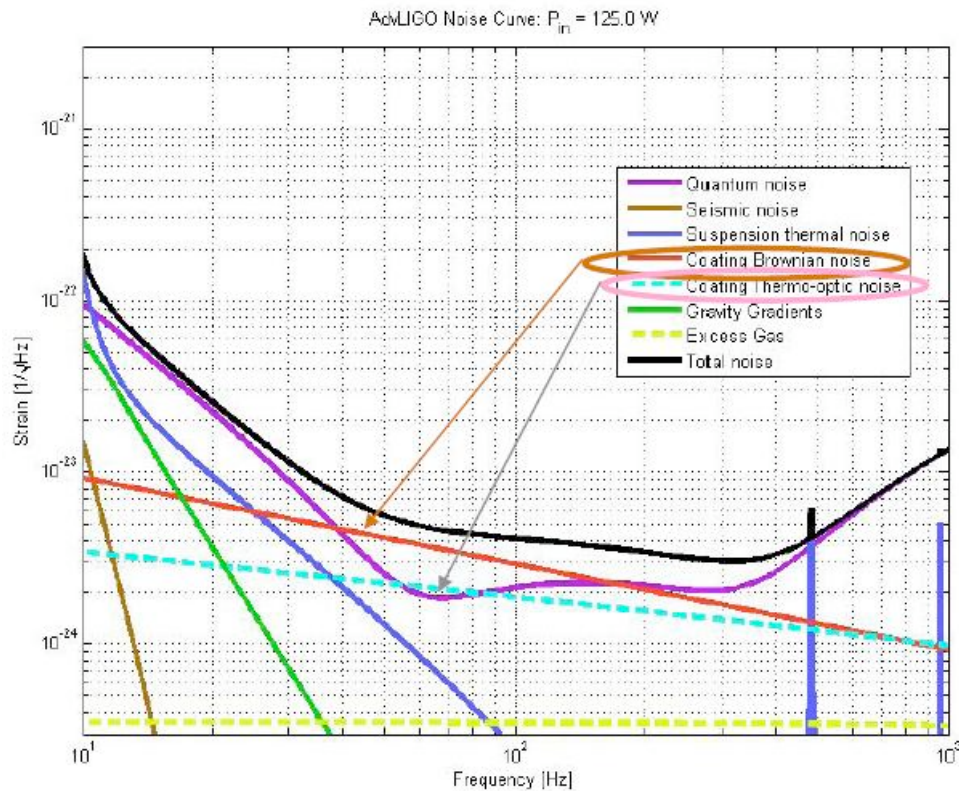


Optimized Mirror Coatings for LIGO

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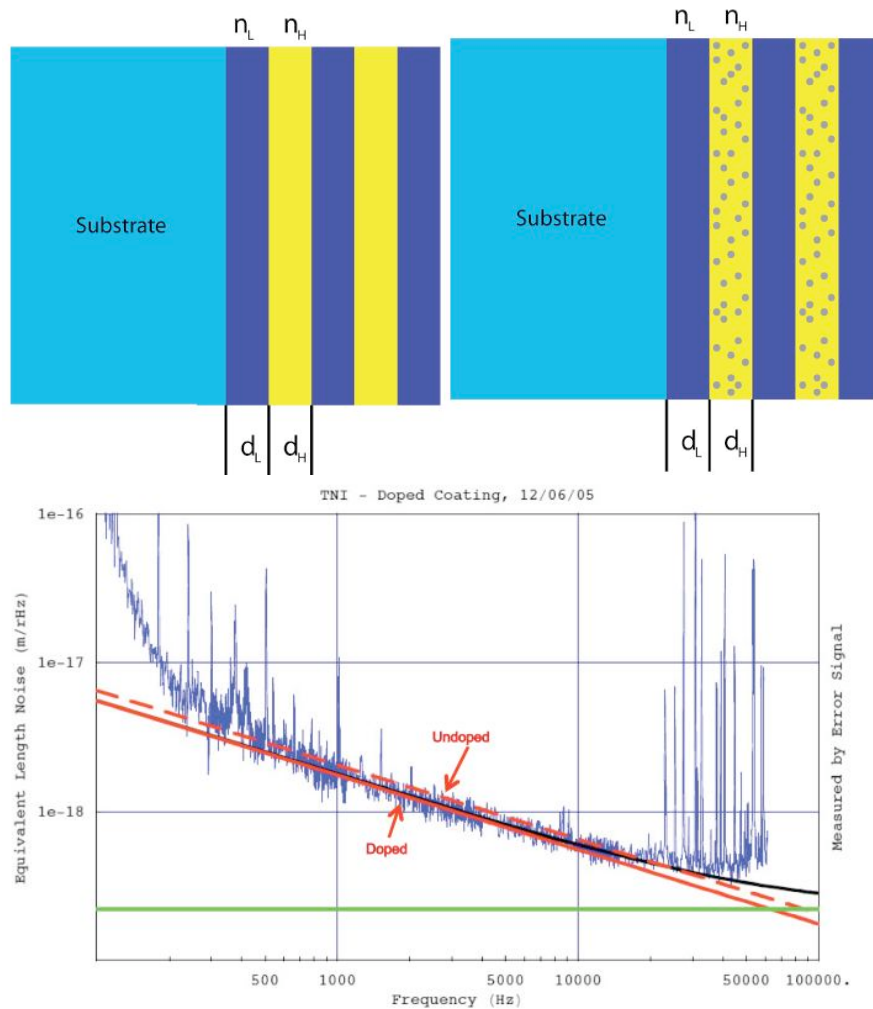
As the many parts that make up the LIGO interferometer are updated and improved for Advanced LIGO, the effect of Brownian Noise caused by the mirror coatings is becoming a more noticeable problem. The chart below shows the expected noise of Advanced LIGO and the amount Brownian noise will contribute to the total noise.



These results are based on the performance of the current mirror coatings, which are made up of alternating quarter-length layers of high and low refractive materials.

The layout of the mirror coatings used in LIGO is used because it produces the best reflectivity. The coating is composed of alternating layers of Tantalum (high index) and Silica (low index); both were chosen for their relatively low mechanical loss. Although this quarter wavelength set-up yields the best reflectance for a minimum number of layers, it does not

produce the minimum amount of noise (1). Most of the noise is caused by the Tantalum. A proposed and moderately successful solution is to dope the Tantalum with Titania (3).



The images above demonstrate this solution, and the graph below the images shows the experimental results. A change can be seen, but it is only a slight improvement.

Another proposed solution is to change the length of the alternating layers, decreasing the width of the Tantalum layers and increasing the width of the Silica. Although straying from the quarter-wavelength form reduces reflectivity, an 'optimized coating' can be found that has the lowest noise for a desired value of reflectance. Because the quarter wavelength coating produces

the best reflectance for a set number of layers, more layers need to be added as the layer thickness is altered to keep reflectance as high as possible (3).

To find this optimal coating, we need a way of calculating reflectance and Brownian Noise. The equation for reflectance is:

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}.$$

To find reflectance, we must first find the electric and magnetic fields using the propagation matrix:

$$\begin{bmatrix} E_i \\ H_i \end{bmatrix} = \begin{bmatrix} \cos \delta_i & jn_i^{-1} Z_0 \sin \delta_i \\ jn_i Z_0^{-1} \sin \delta_i & \cos \delta_i \end{bmatrix} \begin{bmatrix} E_{i+1} \\ H_{i+1} \end{bmatrix} \quad (6)$$

The impedance of the vacuum, (Z_0), equals $\sqrt{(\epsilon_0/\mu_0)}$. n_i is the refractive index, and $\delta_i = k_i l_i$ is the phase thickness. Below is a diagram that physically shows what the propagation matrix is finding.

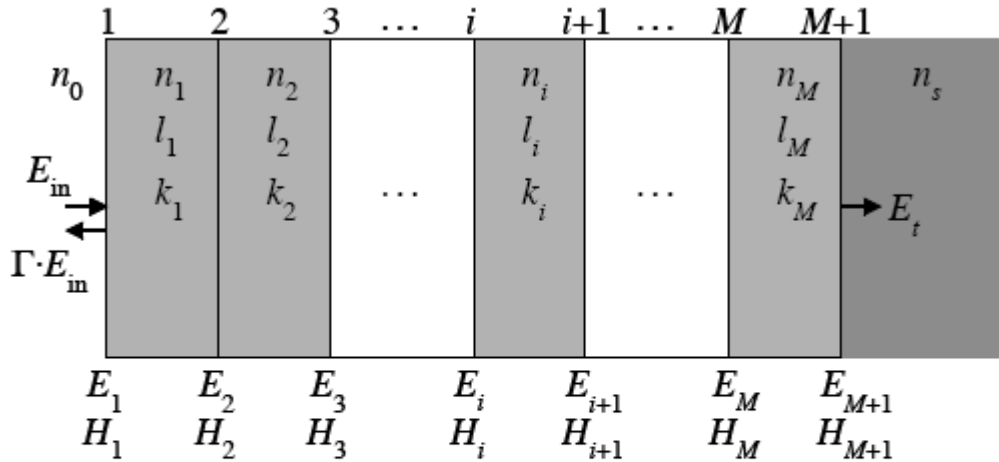


FIG. 5: Diagram of a M layer coating. l_i is the the thickness of the i^{th} layer and n_i is its index of refraction. E_i and H_i are the electric and magnetic fields, respectively, at the i^{th} interface.

The input impedance, $Z_{in} = E_1/H_1$, can be found by chain multiplying the propagation matrix as seen below (3):

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = \mathbf{M}_1 \times \mathbf{M}_2 \times \dots \times \mathbf{M}_M \begin{bmatrix} E_{M+1} \\ H_{M+1} \end{bmatrix}$$

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = \mathbf{M}_1 \times \mathbf{M}_2 \times \dots \times \mathbf{M}_M \begin{bmatrix} 1 \\ n_s/Z_0 \end{bmatrix} E_t.$$

Now we can use the equation $\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$ to find a theoretical value for reflectance.

To find the optimal coating, the relationship between the reflectance and the coating noise, or the Brownian noise, needs to be studied. To find the Brownian noise of a mirror, we have the equation (3):

$$S_B(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1 - \sigma_s^2)}{w Y_s} \phi_{eff}$$

k_B is Boltzmann's constant, T is the temperature, w is the half-width of the Gaussian laser beam, σ_s is the Poisson ratio, and Y_s is the Young's modulus of the substrate. A simplified way to find ϕ_{eff} is to say $\phi_{eff} \approx \phi_c = b_L d_L + b_H d_H$ where $d_{L,H}$ is the total thickness of all the low and high index layers, and $b_{L,H}$ is represented by the equation (3):

$$b_{L,H} = \frac{\lambda_0}{\sqrt{\pi} w} \frac{\phi_{L,H}}{n_{L,H}} (Y_{L,H}/Y_s + Y_s/Y_{L,H}).$$

Y is Young's modulus and ϕ is the loss angle of each substance. There is a significant amount of uncertainty in the calculation of the loss angle of thin layers. The ratio of b_H/b_L is somewhere between 5 and 10 (3).

Now that we have equations for both reflectance and noise, we can study the relationship between the two (3).

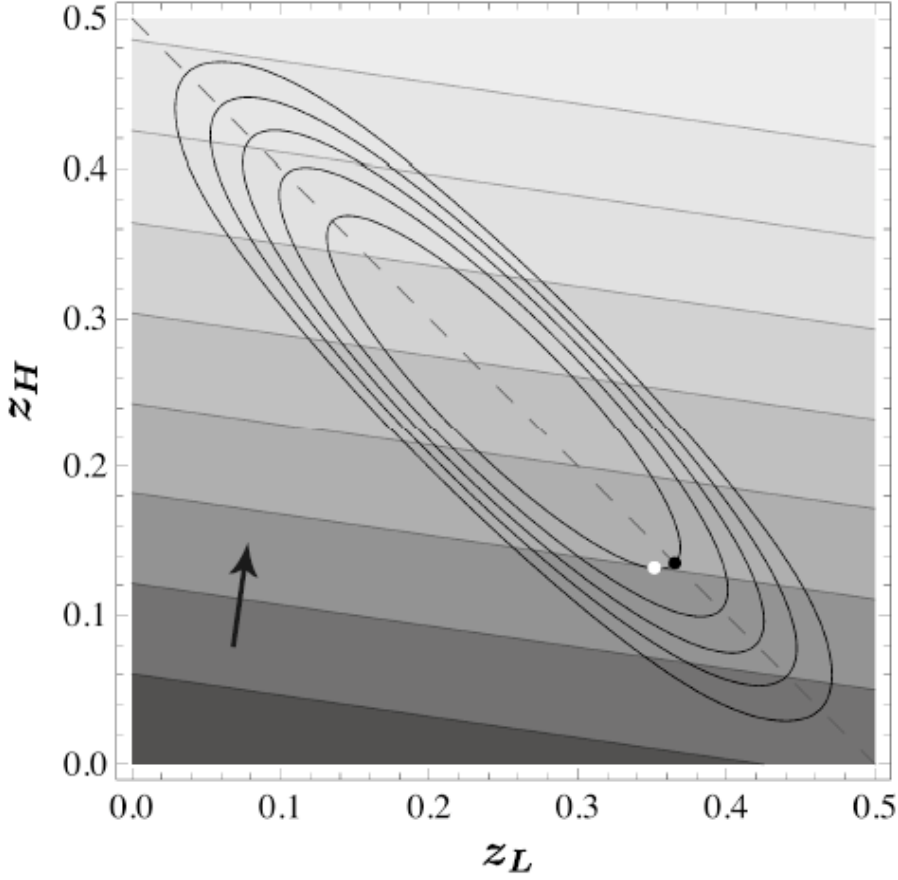


FIG. 6: Contours of constant reflectance (ellipse-like curves) and thermal noise (shaded lines) for a stack of four identical tantala/silica doublets as functions of the silica, z_L , and tantala, z_H , layer thicknesses (in units of local wavelength). The arrow indicates the direction of increasing thermal noise. In this plot $\gamma = 7$. The dashed line consists of points where $z_L + z_H = 0.5$. The white dot marks the point of minimum thermal noise on that particular reflectance contour. The black dot marks a point on the same reflectance contour that satisfies the condition $z_L + z_H = 0.5$. This point has slightly greater thermal noise than the minimum noise point but the difference becomes negligible as the number of doublets (here 4) increases.

Next, the amount of variables is cut down by focusing on doublets that have lengths equal to $\lambda/2$,

as shown by the dotted line in the graph above. This allows us to play with the number of doublets and the thicknesses of the individual silica and tantala layers. Using the above equations to test various values of N after altering the thickness of each layer produce the following results (3):

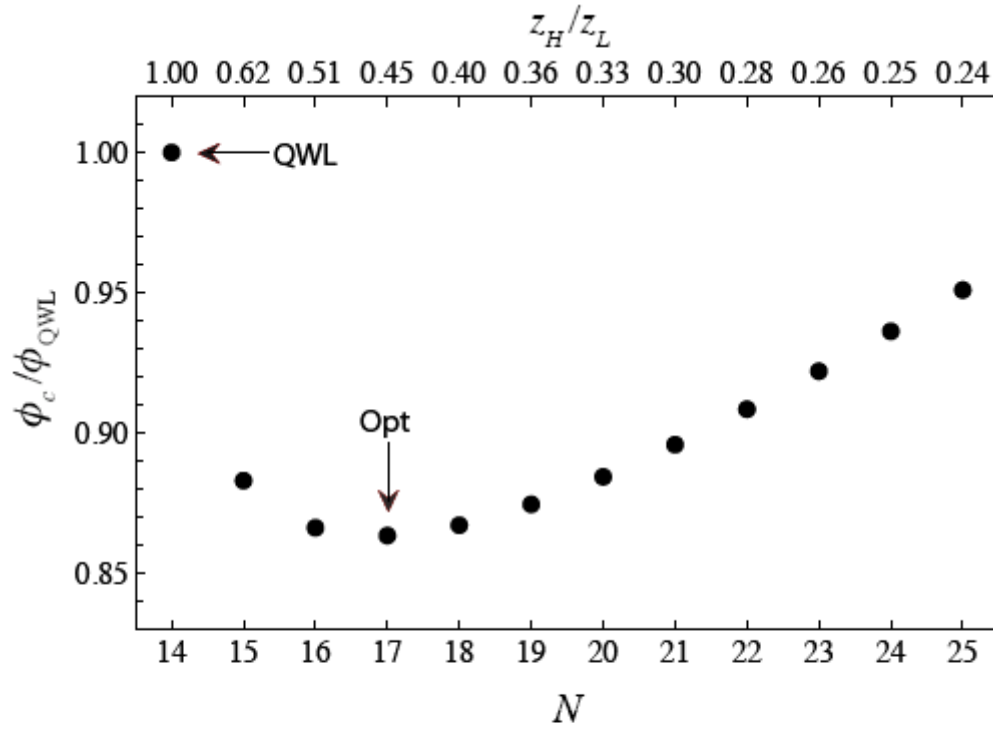


FIG. 7: Plot of the coating loss angle (normalized by the loss angle of the QWL coating) as a function of the number of doublets, N . The corresponding values of the ratio z_H/z_L are shown at the top of the plot. The QWL and the minimum noise (optimized) coatings are indicated in the plot.

We started with (3):

Reference QWL

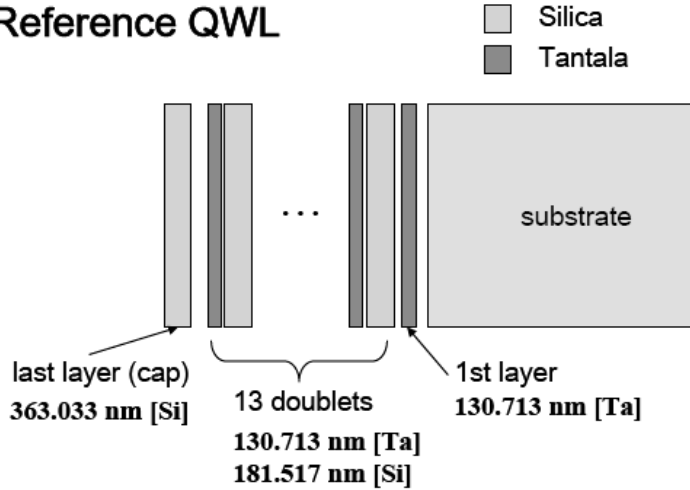


FIG. 8: The structure of the standard quarter wavelength

Now we have (3):

Optimized Prototype

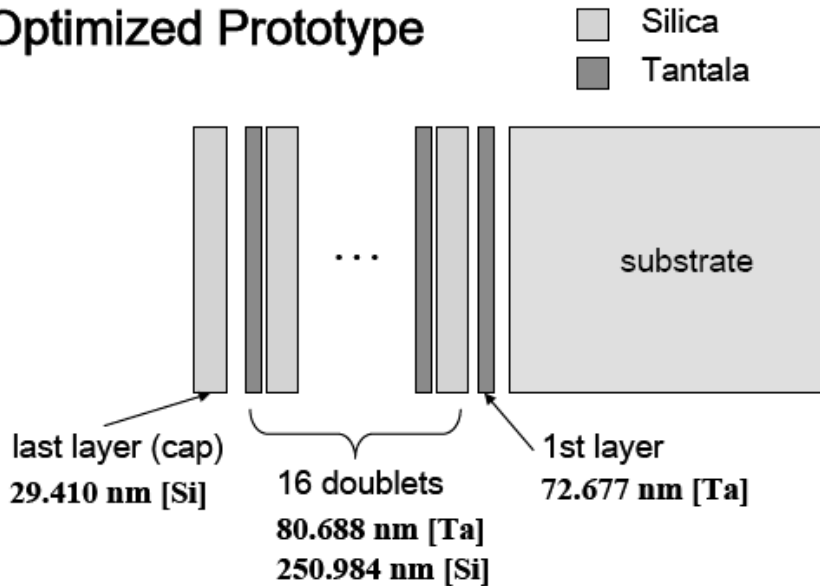
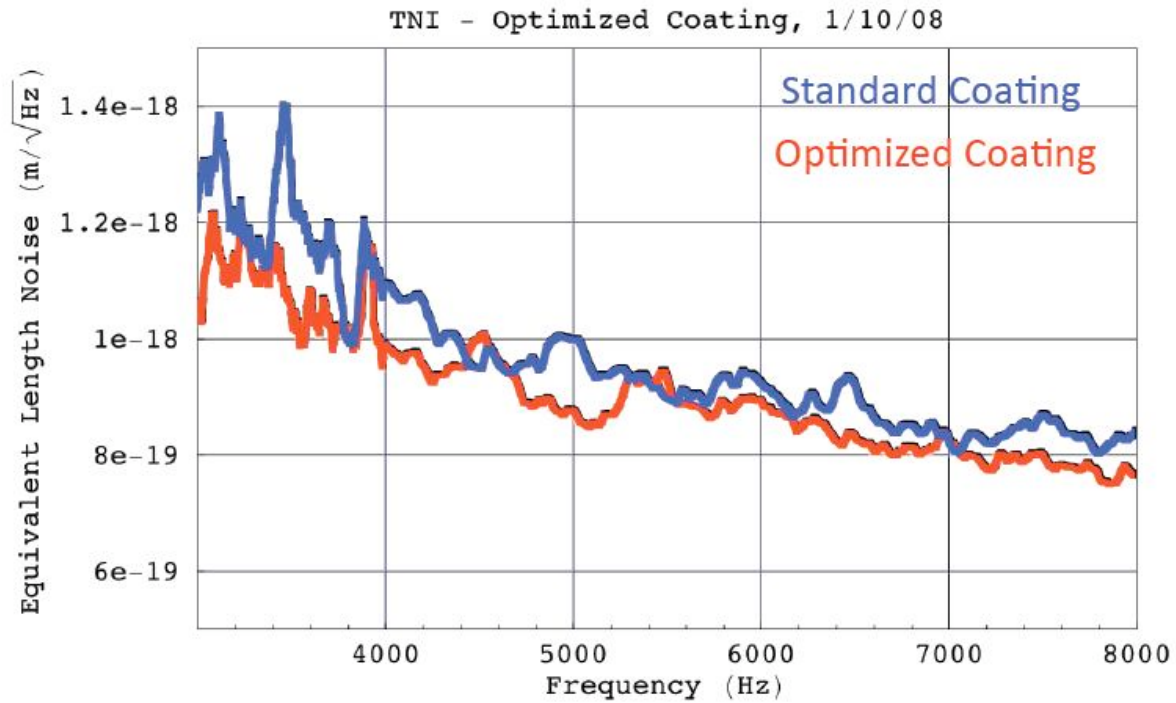


FIG. 9: This is the structure of the optimized coating. This coating was designed for minimal Brownian noise and a transmission of 278 ppm.

So now, instead of alternating quarter-wavelength layers of Silica and Tantalum, we have thicker layers of Silica alternating with thinner layers of Tantalum doped with Titania. When this new coating was experimentally tested, these were the results:



Again, the results can be visually observed on the graph, but the improvement is not drastically significant. Although the alterations produce some noise reduction, there is a lot of current and proposed research that attempts to make a bigger reduction in the Brownian noise.

Recently, genetic algorithms have been used to predict the resulting noise when some of the variables are altered. These tools are capable of analyzing non-differential equations such as the equations currently in use for finding the optimal mirror coating. They also allow for both continuous (layer thickness) and discrete (number of layers) variables. Below are the results for the simplest case where the number of layers (N) is fixed. The reflection coefficient is maximized and there is a given upper bound for the amount of tantala in the coating (1).

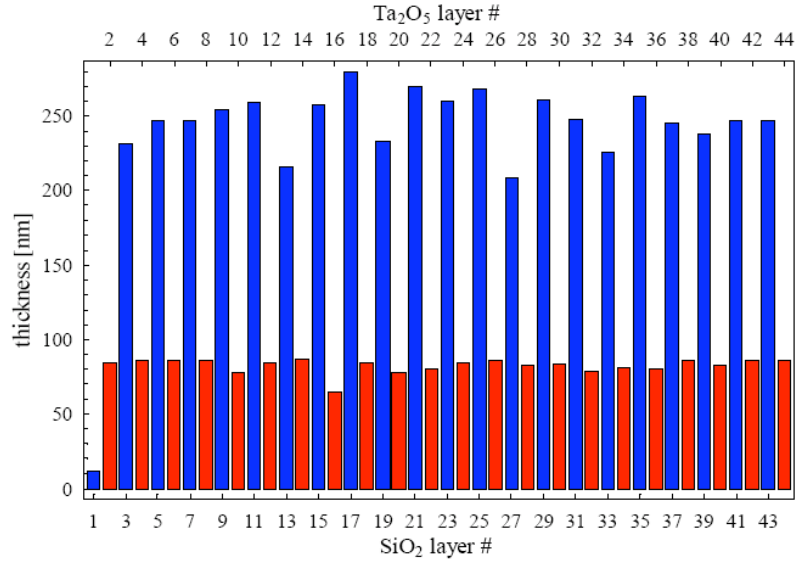


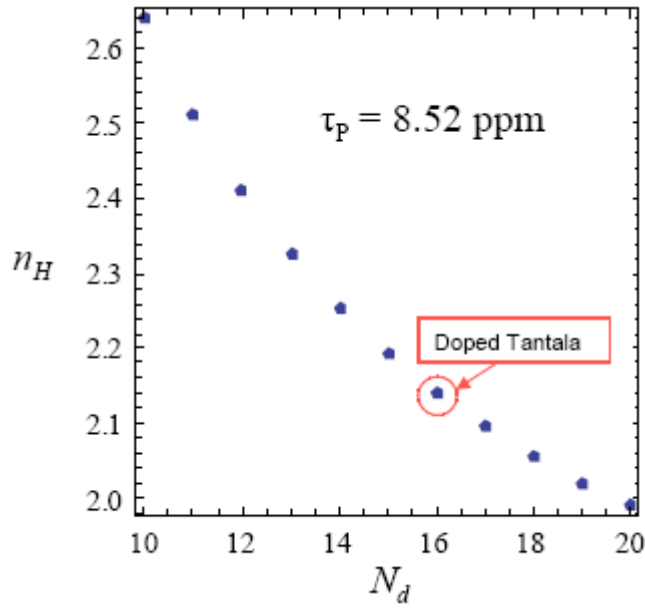
Figure 1. Genetically engineered mirror prototype (best design after 10^5 GA generations). Design goals: Transmissivity $1-|r|^2 \leq 15 \text{ ppm}$; tantalum thickness $\leq 80\%$ of the amount used in standard QWL design with closest reflectivity.

The results of this show that as the number of generations increases, optical lengths of the doublets all seem to be around half a wavelength, with the exceptions lying mostly within the terminal layers. The results also suggested that the reflectivity is best when more tantalum is used (1).

There are currently a few ideas on how to further this research and continue to optimize the relationship between noise reduction and reflectivity. One of these ideas is to use sub-wavelength layers that can be treated as a single homogeneous layer. This could be considered a type of mixture and be used instead of the current high index layers, the best to this point being Tantalum doped with Titania. By using thinner layers, the Titania is less likely to crystallize, allowing us to use more of it. The equations for such a mixture with alternating layers of only two materials are listed below, as well as a graph demonstrating the relationship between the number of total layers (N) and the index of refraction for the higher index material (n_H) (2).

$$n_H = [rn_1^2 + (1 - r)n_2^2]^{1/2}$$

$$\phi_c = \frac{\lambda_0}{\pi^{1/2}w_0} \left[\frac{\phi_L}{n_L} \left(\frac{Y_L}{Y_s} + \frac{Y_s}{Y_L} \right) z_L + \frac{\phi_{H_1}}{n_{H_1}} \left(\frac{Y_{H_1}}{Y_s} + \frac{Y_s}{Y_{H_1}} \right) (r \cdot z_H) + \frac{\phi_{H_2}}{n_{H_2}} \left(\frac{Y_{H_2}}{Y_s} + \frac{Y_s}{Y_{H_2}} \right) (1 - r) \cdot z_H \right],$$



Y_i , and $\square_i$ are material properties. If the low index material is held constant and the desired value for the power transmittance (τ_p) is selected, then the needed value for n_H can be found for N doublets. Then we need to select two materials with n_1 and n_2 and calculate the value of the thickness fraction, r_i , needed to produce the desired value of n_H . Using this value along with the material properties, the theoretical loss angle of the coating can be calculated (2).

Below are the physical properties of some materials that could be used for coatings and the theoretical results of using these materials. Silica is still used as the low-index layer in these calculations (2).

- Silica: $n = 1.46$; $\phi = 4 \cdot 10^{-5}$; $Y = 72 \cdot 10^9$;
- Tantala: $n = 2.09$; $\phi = 4 \cdot 10^{-4}$; $Y = 140 \cdot 10^9$;
- Titania: $n = 2.29$; $\phi = 1.2 \cdot 10^{-4}$; $Y = 165 \cdot 10^9$;
- TiO₂ ::Ta₂O₅ : $n = 2.12$; $\phi = 2 \cdot 10^{-4}$; $Y = 140 \cdot 10^9$ (?)

Reference coating:

16 QWL doublets, Silica + (amorphous) Titania doped Tantala

Materials used in sandwich										
N_d	n_H	TiO ₂ + SiO ₂			TiO ₂ + Ta ₂ O ₅			TiO ₂ + TiO ₂ ::Ta ₂ O ₅		
		r'	ϕ_{eff}	$\phi_{coat}/\phi_{coat}^{ref}$	r'	ϕ_{eff}	$\phi_{coat}/\phi_{coat}^{ref}$	r'	ϕ_{eff}	$\phi_{coat}/\phi_{coat}^{ref}$
14	2.224	0.904	1.14 10 ⁻⁴	0.578	0.660	2.1 10 ⁻⁴	0.951	0.603	1.5 10 ⁻⁴	0.714
15	2.166	0.823	1.08 10 ⁻⁴	0.593	0.371	3.0 10 ⁻⁴	1.330	0.265	1.8 10 ⁻⁴	0.862
16	2.12	0.754	1.03 10 ⁻⁴	0.610	0.128	3.6 10 ⁻⁴	1.594	0	2.0 10 ⁻⁴	1.000

These results suggest that using alternating sub-wavelength layers of Titania and Silica can cut down almost half the noise of the currently suggested quarter-wavelength set-up of alternating Silica and Titania doped Tantala layers. Although the numbers aren't as impressive, the results also suggest that alternating sub-wavelength layers of Titania and Tantala can perform better than the current coatings (2).

There may be some technological difficulties laying down such thin layers, but the production of sub-wavelength layered coating for X-Ray mirrors suggests the construction of this type of coating is very possible. A benefit of this type of layered mixture is that the lengths of each layer need not be exact because only the ratio of the total amount of each material is important (2).

An additional, less explored area is altering the geometric make-up of the mixtures.

Theoretical calculations have already been done for sub-wavelength layers that are thin enough

to be considered a mixture, straying from the traditional amorphous mixtures that were initially used. Results show that the new layered mixtures will most likely outperform the amorphous mixtures, hinting that there may be even more possibilities in alerting the way substances are mixed or layered (2).

The search for the optimal mirror coating is a slow but continuous process. The future of sub-wavelength layers could potentially be the first drastic improvement since LIGO began, and will hopefully lead to further studies in the geometry involved when mixing the materials that make up the high index layer, and eventually cut Brownian noise down to a level where it will no longer have such an effect on the noise budget.

Sources:

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c Waves Group, Department of Engineering, University of Sannio, I-82100, Benevento, Italy

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