Modal Frequency Analysis of the Cryogenic Payload

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Introduction:

The Roma group of the INFN has been working on the new payload for the end towers of Advanced VIRGO. The new payload is to be situated below the six seismic filters (al., 2001). At the top of the payload is the new marionette reference mass, or MRM. This is the biggest improvement over the old payload. A wire from the last seismic filter supports a junction for the three wires that hold up the MRM. This junction was dubbed the "Chinese hat". The MRM uses four arms with coils attached to control the marionette which has four permanent magnets. The coils in the arms of the MRM work as electromagnets to attract and repel the permanent magnets on the marionette, thereby stabilizing it. Below the MRM is the marionette which serves as a mass to which the mirror and mirror reference mass are attached.

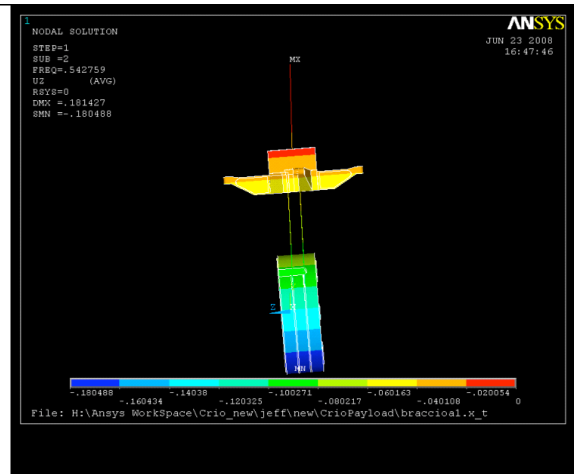
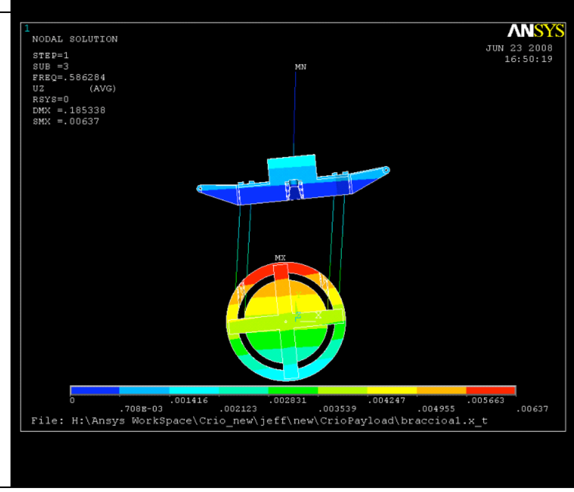
The aforementioned mirror reference mass is like a frame that surrounds the mirror. Like the MRM it has four coils which manipulate four permanent magnets on the mirror to stabilize it. The mirror is the mirror which will be at the end of the north and west arms and will reflect the laser. And finally this entire payload with be in a cryogenic chamber of around 4 degrees Kelvin. The interferometer is so sensitive that even changes caused my thermal fluctuations can result in noise. This should eliminate thermal noise and with the new MRM the payload should also be further isolated from seismic perturbations. (al., 2001)

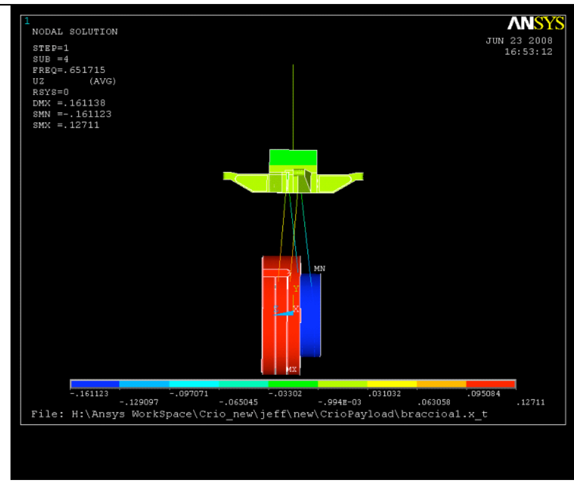
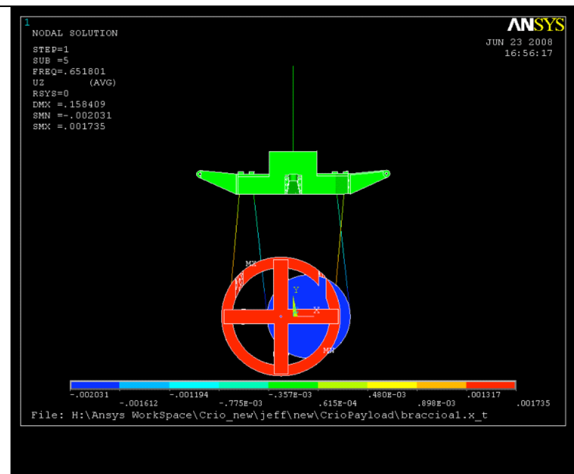
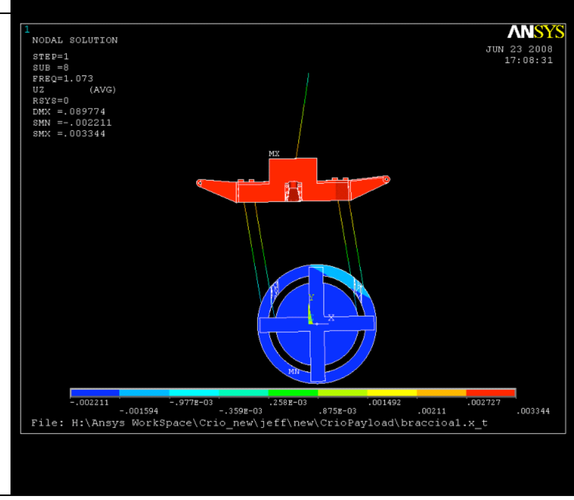
It is important to know the frequencies of the pendulum modes of the payload because they must be outside the sensitivity range of the interferometer. This is a requirement of the payload. The sensitivity range is approximately between 10 Hz to 5000 Hz (al., 2001). The seismic noise in this range must be as low as possible so the natural pendulum frequencies of the payload need to be outside the sensitivity range. The frequencies also need to be known to set up the control system that stabilizes the marionette and mirror. By simulating the payload before it is constructed and implemented the frequencies can be optimized.

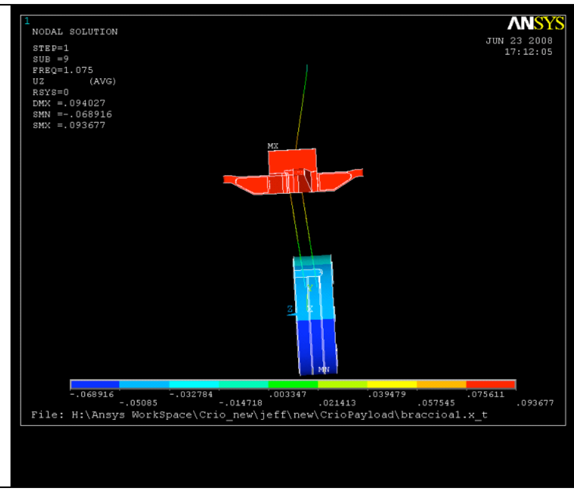
The following is the results of a finite element analysis on the VIRGO cryogenic payload. The software package ANSYS was used to perform the dynamic simulations. With ANSYS one can create a model, then "mesh" the model into many small pieces which the software then uses to solve the differential equations used to find loads, forces, deformations, etc. that the model experiences. Massimo Granata at the University of Rome "La Sapienza" created the model I used. The goal was to find the modal frequencies of the payload and its components and compare them to calculated frequencies. This was done in three different steps. In the first we observed the modes of the payload without the MRM, then just the MRM by itself, and finally the complete payload. In this way we can see the effect that the MRM has on the modes of the payload.

Part One: Payload without the MRM

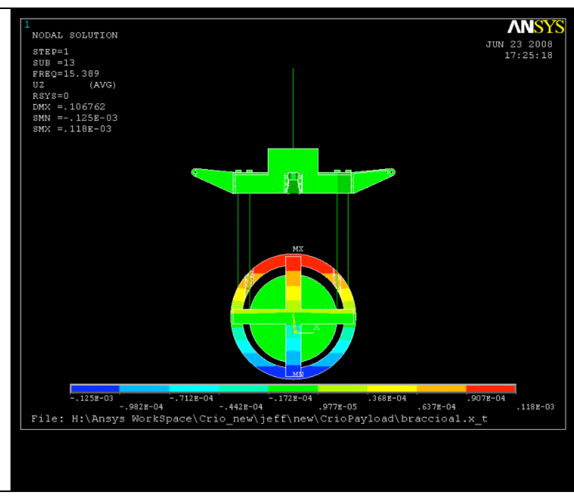
Pendulum Modes

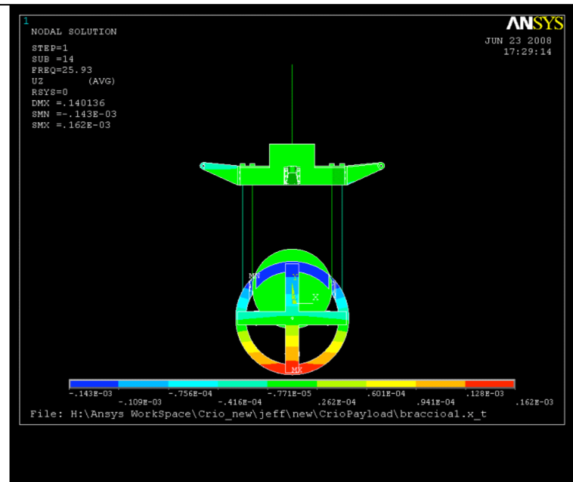
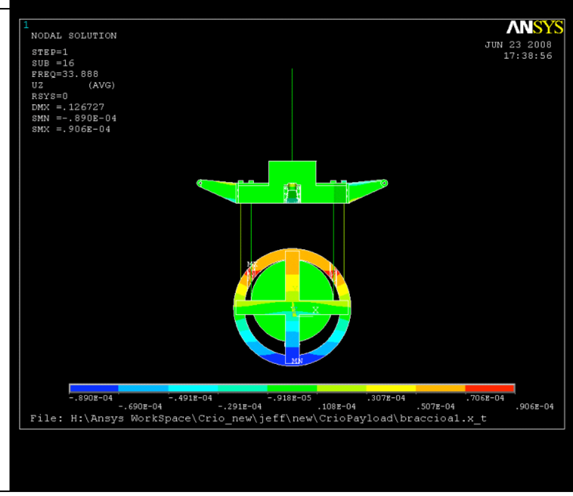
Description	**Frequency (Hz)**	**Calculated Frequency (Hz)**	**Picture**
Pendulum z-axis	0.542759	0.505408	
Pendulum x-axis	0.586284	0.505408	

Mirror pendulum z-axis	0.651715	0.643217	
Mirror pendulum x-axis	0.651801	0.643217	
Pendulum differential x-axis	1.073000	0.944716	

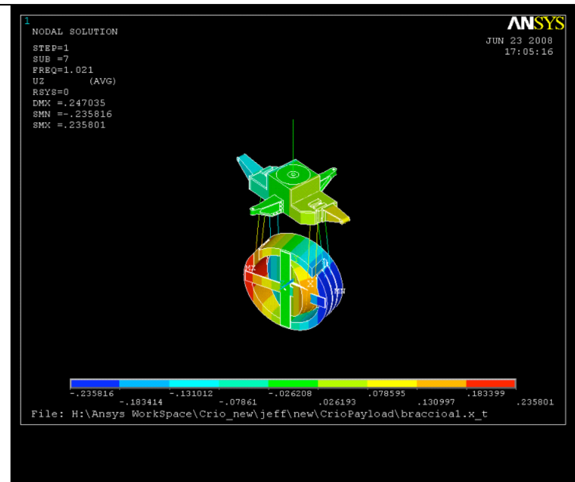
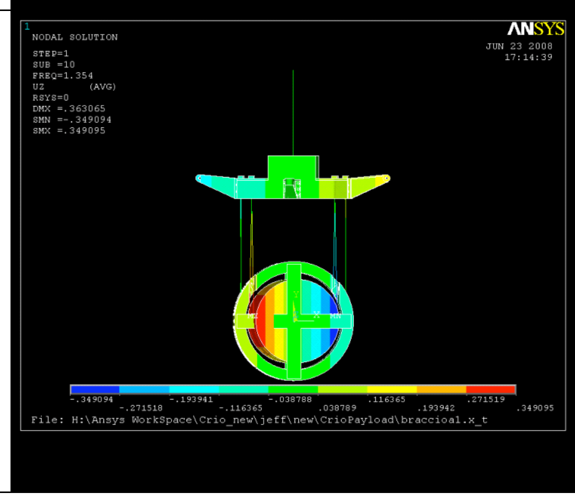
Pendulum differential z-axis	1.075000	0.944716	

Bouncing Modes

Description	**Frequency (Hz)**	**Calculated Frequency (Hz)**	**Picture**
Bouncing y-axis	15.389000	15.516400	

Bouncing differential y-axis	25.930000	21.851300		
Bouncing differential y-axis	33.888000	30.523200		

Torsion Modes

Description	**Frequency (Hz)**	**Calculated Frequency (Hz)**	**Picture**
Rotational differential y-axis	1.021000	1.043080	
Mirror rotational y-axis	1.354000	1.277790	

The calculated frequencies for pendulum and bouncing modes were found using the following matrix:

$$\begin{bmatrix} K_1 + K_2 + K_3 - M_{mario}\,\omega^2 & -K_2 & -K_3 \\ -K_2 & K_2 - M_{mirror}\,\omega^2 & 0 \\ -K_3 & 0 & K_3 - M_{mrif}\,\omega^2 \end{bmatrix}$$

where

Mass of the marionette: $M_{mario} = 117\ kg$

Mass of the mirror: $M_{mirror} = 20\ kg$

Mass of the mirror reference mass: $M_{mrif} = 31.8\ kg$

For pendulum modes:

$$K_1 = \frac{M_{mario} * g}{L_{mario}}$$

Where $= 9.8\ \frac{m}{s^2}$, $L_{mario} = 0.4505\ m$.

K2 and K3 are similar for the mirror and mirror reference mass using their respective masses and $L_{mirror} = L_{mrif} = 0.600\ m$.

For bouncing modes:

$$K_1 = \frac{Y_{mario} * S_{w\,mario}{}^2}{L_{mario}}$$

Where the Young's Modulus $Y_{mario} = 116\ GPa$, and the wire section $S_{w\,mario} = 3 * 10^{-3}\ m$.

Once again K_2 and K_3 are similar for the mirror and mirror reference mass using their Young's Modula $Y_{mirror} = Y_{mrif} = 69\ GPa$, and wire sections $S_{w\,mirror} = S_{w\,mrif} = 1 * 10^{-3}\ m$.

For torsion modes:

The masses M_{mario}, M_{mirror}, and M_{mrif} in the matrix must be replaced by the moment of inertia of the components about the vertical axis y.

$I_{mario} = 1.66790$

$I_{mirror} = 0.19033$

$I_{mrif} = 0.74794$

$$K_1 = \frac{I_2 * G_1}{L_{mario}}$$

Where

$$G_1 = \frac{Y_{mario}}{2(1 + \sigma_{mario})}$$

The Poisson's ratio $\sigma_{mario} = 0.32$.

$$I_2 = \frac{1}{2}\pi\, r_{mario}^4$$

The radius of the marionette's wire $r_{mario} = 1.5 * 10^{-3}\ m$.

$$K_2 = \frac{M_{mirror}\ g\ R_{mirror}^2}{L_{mirror}}$$

The radius of the mirror $R_{mirror} = 0.175\ m$.

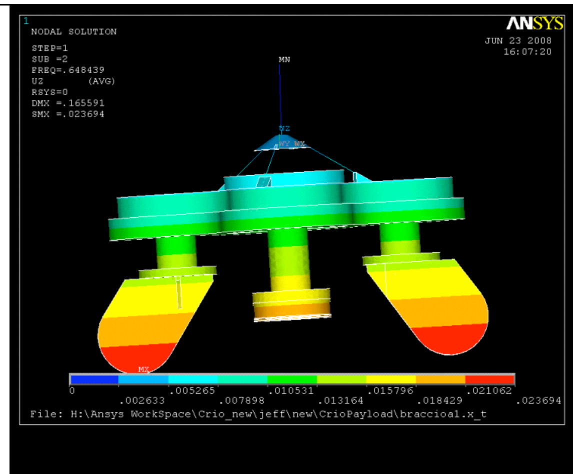
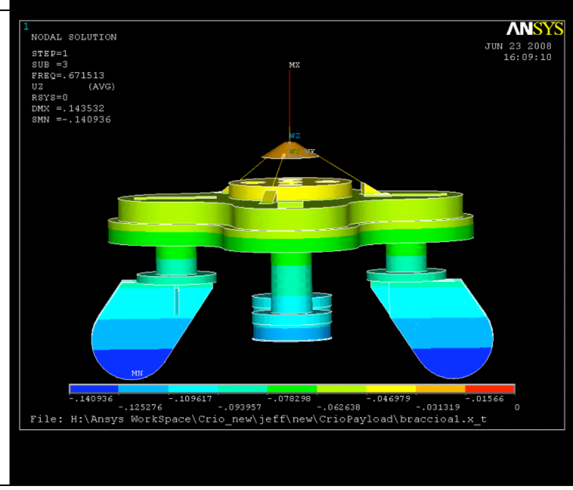
K_3 is similar to K_2 with its respective mass, wire length, and $R_{mrif} = 0.22\ m$.

Conclusion:

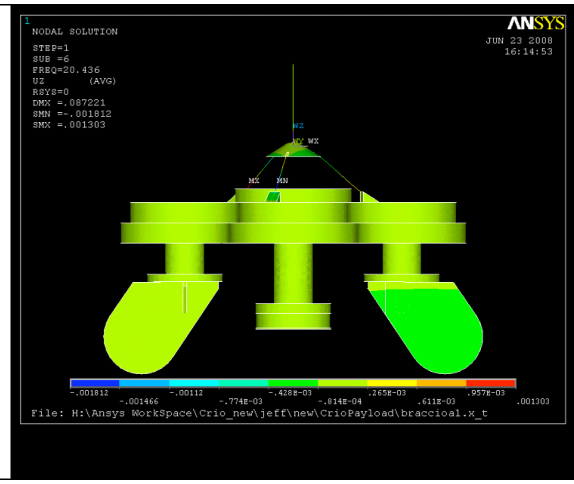
One can solve for the modal frequencies ω by setting the matrix's determinant equal to zero and solving for ω. The reported frequencies $= \frac{\omega}{2\pi}$. As we can see by using these calculations there is a strong correlation between the theoretical modal frequencies and the frequencies as determined by ANSYS. The only large discrepancy between the ANSYS reported frequencies and the calculated frequencies was for the bouncing modes. I believe this is because the calculations did not take into account that the mirror and mirror reference mass are suspended by four wires each.

Part Two: The MRM

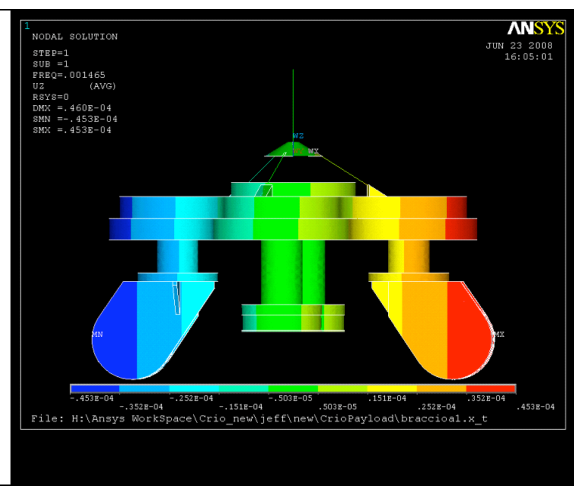
Pendulum Modes

<table>
<tr><th>Description</th><th>Frequency (Hz)</th><th>Calculated Frequency (Hz)</th><th>Picture</th></tr>
<tr><td>Pendulum x-axis</td><td>0.648439</td><td>0.899216</td><td>
</td></tr>
<tr><td>Pendulum z-axis</td><td>0.671513</td><td>0.899216</td><td>
</td></tr>
</table>

Bouncing Mode

Description	Frequency (Hz)	Calculated Frequency (Hz)	Picture
Bouncing y-axis	20.436000	22.534400	

Torsion Mode

Description	Frequency (Hz)	Calculated Frequency (Hz)	Picture
Rotational y-axis	0.062855	0.0712217	

Here more simple calculations can be used than in Part One. We can treat the MRM as a mass on the end of a string.

For pendulum modes:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{L_{mrm}}}$$

Where $L_{mrm} = 0.307\ m$.

For bouncing modes:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{\pi\, r_{mrm}^2\, Y_{mrm}}{L_{mrm}\, M_{mrm}}}$$

Where the radius of the MRM's wire $r_{mrm} = 1.5 * 10^{-3}\ m$, the Young's Modulus $Y_{mrm} = 116\ GPa$, and the MRM's mass $M_{mrm} = 133.23\ kg$.

For torsion modes:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{K_{mrm}}{I_{mrm}}}$$

Where $I_{mrm} = 11.367$.

$$K_{mrm} = \frac{2\, I_2\, G}{L_{mrm}}$$

$$I_2 = \frac{\pi}{2}\, r_{mrm}^4$$

$$G = \frac{Y_{mrm}}{2(1 + \sigma_{mrm})}$$

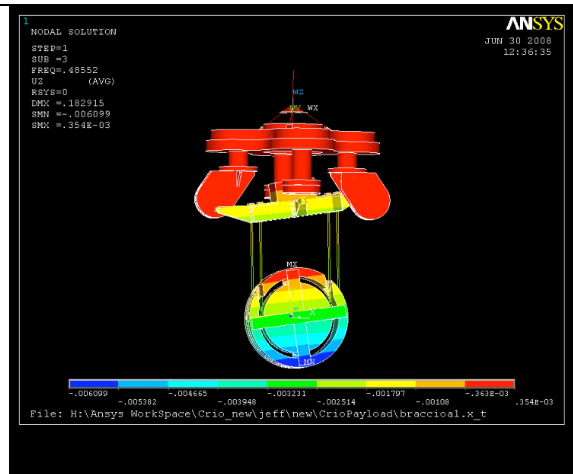
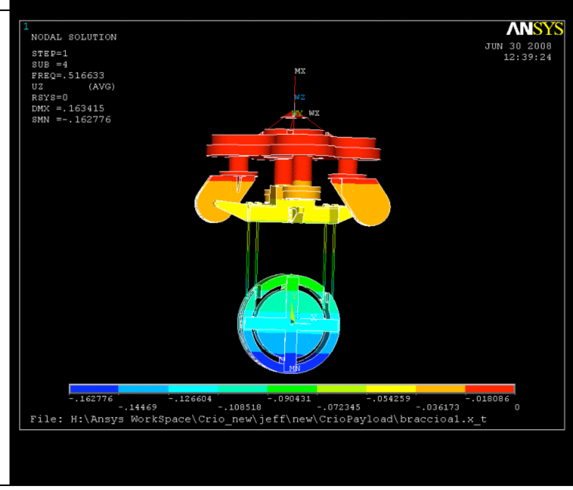
Where $\sigma_{mrm} = 0.32$.

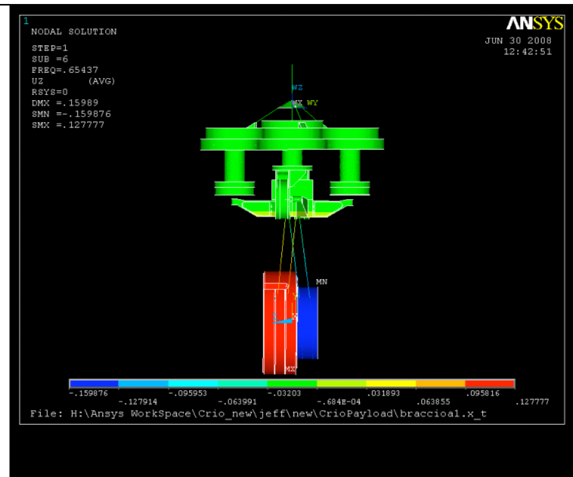
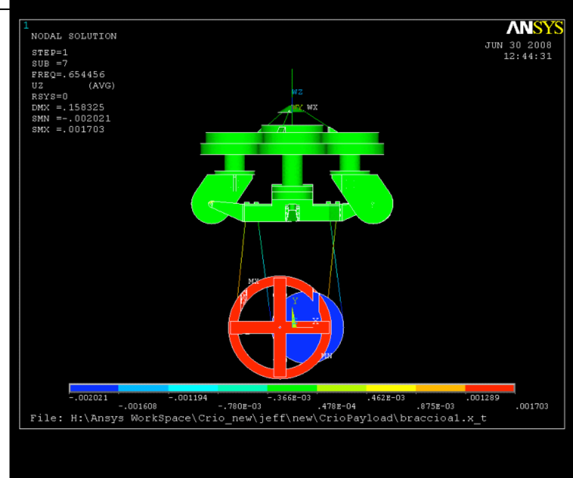
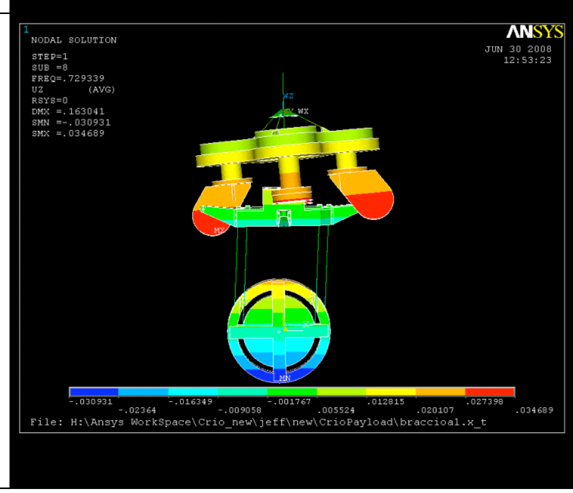
Conclusion:

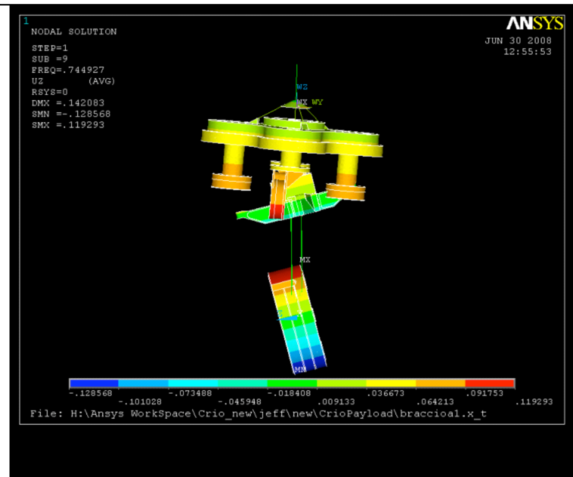
Once again we observe the strong correlation between the theoretical frequencies and the frequencies reported from ANSYS.

Part Three: The Complete Payload

Pendulum Modes

Description	Frequency (Hz)	Calculated Frequency (Hz)	Picture
Marionette and mirror pendulum x-axis	0.485520	0.384807	
Marionette and mirror pendulum z-axis	0.516633	0.384807	

Mirror pendulum z-axis	0.654370	0.643217	
Mirror pendulum x-axis	0.654566	0.643217	
Pendulum differential x-axis	0.729339	0.764725	

<table>
<tr><td>Pendulum differential z-axis</td><td>0.744927</td><td>0.764725</td><td></td><td>
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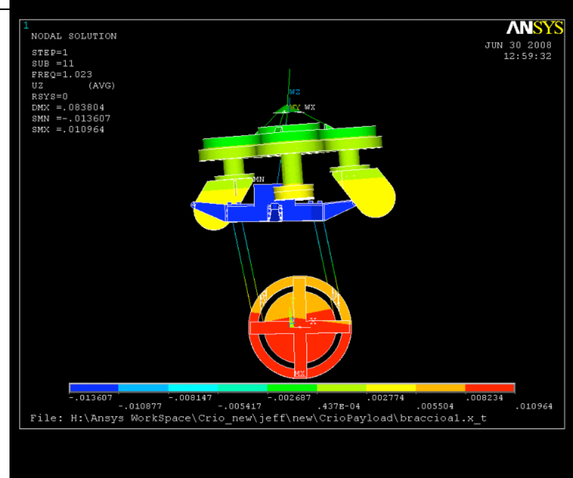
<tr><td>Pendulum differential x-axis</td><td>1.023000</td><td>1.204120</td><td></td><td>
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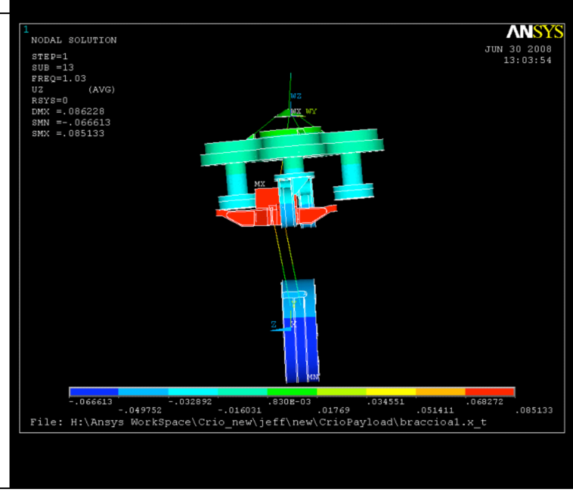
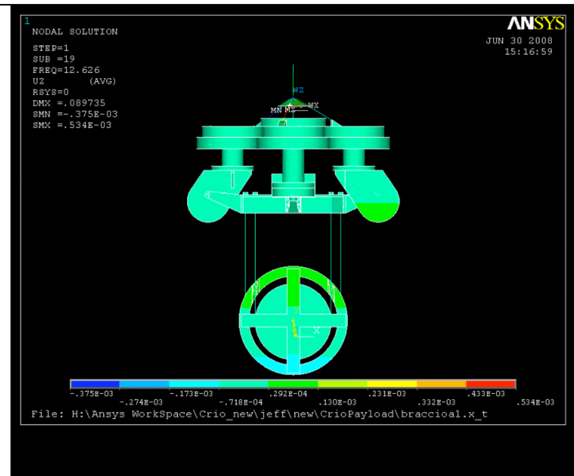
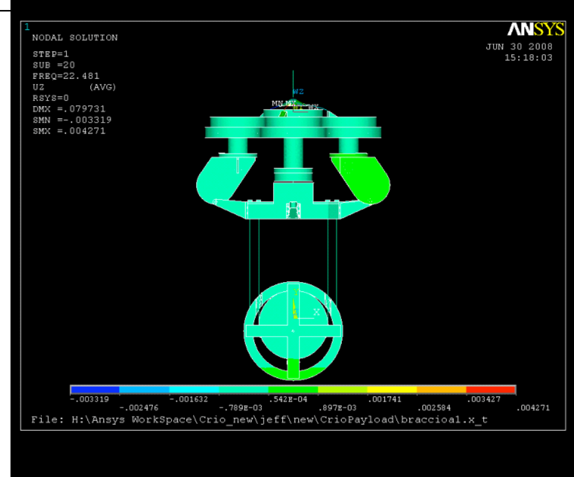
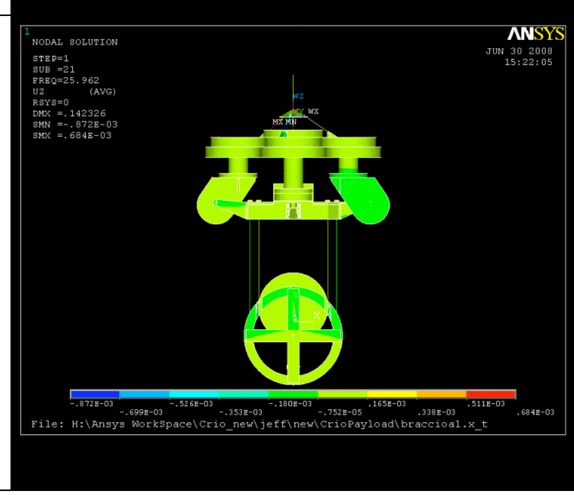
<tr><td>Pendulum differential z-axis</td><td>1.030000</td><td>1.204120</td><td></td><td>
</td></tr>
</table>

Bouncing Modes

Description	**Frequency (Hz)**	**Calculated Frequency (Hz)**	**Picture**
Bouncing y-axis	12.626000	12.457500	
Bouncing differential y-axis	22.481000	20.994100	
Mirror bouncing y-axis	25.962000	26.549600	

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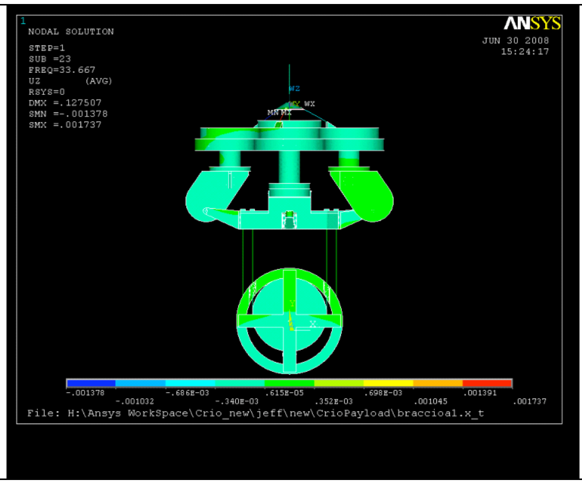
<tr><td>Bouncing differential y-axis</td><td>33.667000</td><td>37.897600</td><td>
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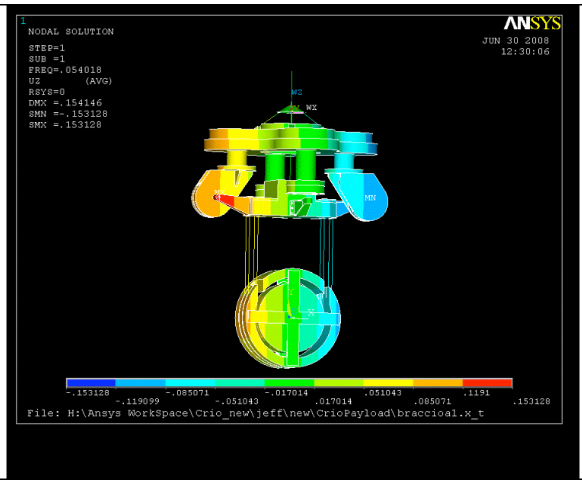
Torsion Modes

<table>
<tr><th>Description</th><th>Frequency (Hz)</th><th>Calculated Frequency (Hz)</th><th>Picture</th></tr>
<tr><td>Torsion y-axis</td><td>0.054018</td><td>0.043974</td><td>
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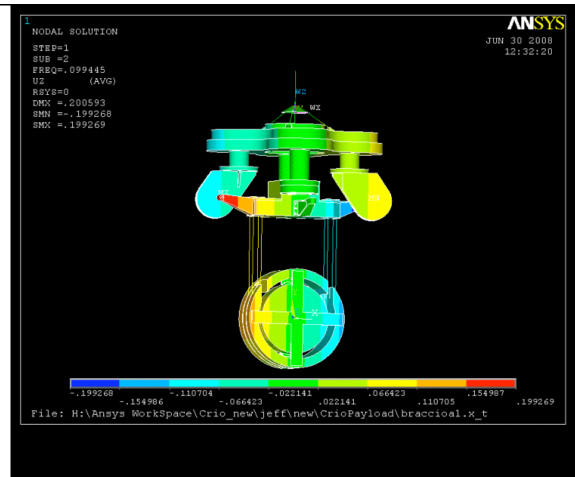
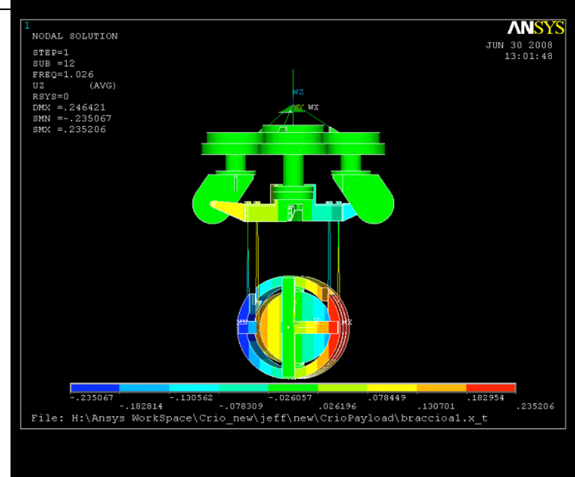
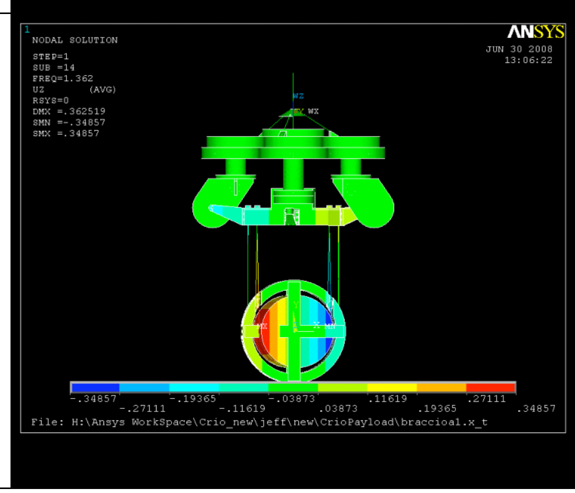
Torsion differential y-axis	0.099445	0.099144	
Marionette and mirror torsion y-axis	1.026000	1.043080	
Mirror torsion y-axis	1.362000	1.277800	

The following matrix was used for the calculation of pendulum and bouncing frequencies for the complete payload:

$$\begin{bmatrix} K_1 + K_2 - M_{mrm}\,\omega^2 & -K_2 & 0 & 0 \\ -K_2 & K_2 + K_3 + K_4 - M_{mario}\,\omega^2 & -K_3 & -K_4 \\ 0 & -K_3 & K_3 - M_{mirror}\,\omega^2 & 0 \\ 0 & -K_4 & 0 & K_4 - M_{mrif}\,\omega^2 \end{bmatrix}$$

Where M_{mrm}, M_{mario}, M_{mirror}, and M_{mrif} are as previously defined.

For pendulum modes:

$$K_1 = \frac{M_{mrm} * g}{L_{mrm}}$$

K_2, K_3, and K_4 are similar with the mass and wire length replaced with the respective properties of the marionette, mirror, and mirror reference mass.

For bouncing modes:

$$K_1 = \frac{Y_{mrm} * {S_{w\,mrm}}^2}{L_{mrm}}$$

K_2, K_3, and K_4 are similar with the MRM's properties replaced by the properties of the marionette, the mirror, and the mirror reference mass, respectively.

For torsion modes:

Once again the masses in the matrix must be replaced with the moments of inertia about the y-axis.

$$K_1 = \frac{I_{2\,mrm} * G_{1\,mrm}}{L_{mrm}}$$

Where

$$G_{1\,mrm} = \frac{Y_{mrm}}{2(1 + \sigma_{mrm})}$$

$$I_{2\,mrm} = \frac{1}{2}\pi\, r_{mrm}^4$$

K_2 is similar with the MRM's properties replaced by the marionette's properties.

$$K_3 = \frac{M_{mirror}\, g\, R_{mirror}^2}{L_{mirror}}$$

K_4 is similar to K_3 with the mirror's properties replaced by the mirror reference mass' properties.

Conclusion:

As with the payload without the MRM, one can solve for the modal frequencies ω by setting the matrix's determinant equal to zero and solving for ω. The reported frequencies $= \frac{\omega}{2\pi}$. As we can see by using these calculations there is a strong correlation between the theoretical modal frequencies and the frequencies as determined by ANSYS. The only noticeably large discrepancy between the calculated frequency and the frequency from ANSYS is for the last bouncing mode. I believe this is because the calculation did not take into account the three wires that support the MRM and that the mirror and mirror reference mass are suspended by four wires each.

For the pendulum modes we can observe that essentially the complete payload combines the modal frequencies of the payload with the MRM and the MRM by itself. But because of the geometry the modes of the MRM affect the rest of the payload. Also unique to the complete payload were the modes that involved the movement of the MRM and the mirror and mirror reference mass while the marionette remained stationary.

The torsion modes of the complete payload include the same torsion modes as the payload without the MRM. However the torsion mode of the MRM by itself was replaced by the mode and differential mode of the entire payload twisting.

The bouncing modes of the complete payload include the two bouncing modes of the mirror that we observed on the payload without the MRM. However the bouncing of the payload without the MRM is replaced by bouncing of the entire payload and the bouncing of just the MRM is replaced by a bouncing differential of the entire payload.

We can conclude that the inclusion of the MRM results in essentially the combination of the modes of the payload without the MRM and the modes of just the MRM. The modes from the MRM by itself usually turn up as the entire payload moving. There are no dramatic effects on the range or values of modal frequencies. Aside from the bouncing modes the pendulum modes for the payload are below the 10 Hz threshold of the VIRGO sensitivity range. However the bouncing modes can be neglected because the six seismic filters above the payload will suppress any effect those modes may have. Therefore the inclusion of the MRM on the payload for VIRGO Advanced will not cause any foreseeable issues at least in terms of the mechanical modal frequencies.

At the time of completion of this report the Roma team is working in the lab at VIRGO on a prototype of the new payload. They are conducting tests on how it performs in the cryogenic chamber at 4 kelvin.

In the future work will be underway to investigate adding two more arms to the MRM for further control of the marionette. Also silicon carbide is being considered as the material to build the MRM from. The current material aluminum has the MRM weighing in at around one hundred kilograms. It would be desirable to reduce this mass.

References

Ballardin, Et al. "Measurement of the VIRGO superattenuator performance for seismic noise suppression." Review of Scientific Instruments 72 (2001): 3643-652.